

Bargaining, Reputation and Competition*

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June 30, 2015

Abstract

This paper addresses the question of whether “playing the tough bargainer” is a useful strategy for rational negotiators in competitive environments. The unique equilibrium outcome of a continuous-time multilateral bargaining game between a single seller and two buyers (i.e., a three-player war of attrition game) show that it does not benefit the buyers—the long side of the market—if the probability of obstinacy is constant and independent of the initial demands. This result is robust in the sense that the buyers’ heterogeneity about their flexibility does not weaken the intensity of the competition. When the seller decides which buyer to negotiate first, he not only chooses his bargaining partner but also picks his outside option. The seller can strengthen his bargaining position against both buyers by picking the unattractive buyer, who is greedier or tougher than the other buyer, first and by leaving the attractive buyer aside because he starts the sequel with a strong outside option. (JEL C72, C78, D43, D83, L13)

1. INTRODUCTION

Negotiators often use various bargaining tactics that are likely to lead to inefficient outcomes. One example of such a tactic is standing firm and not backing down from the last offer. One factor that makes this tactic effective is the negotiators’ uncertainty regarding the adversaries’ commitment (Schelling 1960 and Arrow, Mnookin, Ross, Tversky, and Wilson 1995). A tough bargainer attempts to convince the other parties that he cannot change his offer and make further concessions because he is committed to a particular minimum position (Tedeschi, Schlenker, and

*This research was supported by the Marie Curie International Reintegration Grant (# 256486) within the European Community Framework Programme. I would like to thank David Pearce, Ennio Stacchetti, and Ariel Rubinstein for their helpful comments. I would like to extend my sincerest thanks and appreciation to the Associate Editor and two anonymous referees whose feedbacks immensely improved the paper. All the remaining errors are my own.

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Bonoma 1973). Likewise, a rational negotiator could mimic a tough bargainer and use similar arguments to make his obstinacy credible. An important question is whether “playing the tough bargainer” is a beneficial strategy for rational negotiators or not.

A growing literature shows that commitments lead to inefficient outcomes, and playing the tough bargainer in bilateral negotiations benefits a rational negotiator, especially when he has a reputational advantage.¹ This paper examines if playing the tough bargainer benefits a negotiator in a competitive environment (e.g., housing market, labor market, or car market), where searching and bargaining for a better deal are the key aspects.

The stylized model of this paper is inspired by two key characteristics of these markets. First, the sellers are monopolistically competitive because they sell differentiated products and each seller has a monopoly power on a specific group of buyers. Second, a seller posts a price and the buyers offer their bids, but it is common knowledge between these agents that the initial offers are negotiable.² Rather than making a market equilibrium analysis, I focus on the strategic interaction between a single seller and two buyers, and investigate how commitments affect these players’ pricing, negotiation and search behavior.

There are three defining features of the model. First, a single seller negotiates with two buyers over the sale of one item. Second, the buyers compete in a Bertrand fashion and make initial posted-price offers simultaneously. The seller can accept one of these offers costlessly, or else visit one of the buyers and try to bargain for a higher price.³ Third, each of the three players suspects that his opponents might have some kind of commitment forcing them to insist on their initial offers. That is, each player would be a commitment/obstinate type with some positive probability. This uncertainty provides incentive to the flexible (or rational) seller to build a reputation for obstinacy by being tough against one buyer and to convince the other buyer that he is indeed tough. Obstinate types take an extremely simple form. Following the “crazy” types of Kreps and Wilson (1982) and Milgrom and Roberts (1982), and parallel to the “ r -insistent” type of Myerson (1991) and Abreu and Gul (2000), an obstinate player always demands a fixed share and accepts an offer if and only if it weakly exceeds that share.

¹See, for example, Shelling (1960), Crawford (1982), Myerson 1991, Muthoo (1996), Kambe (1999), Abreu and Gul (2000), and Ellingsen and Miettinen (2008). Ozyurt (2015) provides a detailed discussion about this literature.

²A house owner or a realtor who wants to sell his estate usually advertises the property on various websites, such as Zillow, Hotpads, or Craigslist, with some pictures and a listing price. Interested buyers then make counteroffers. The homeowner/realtor either accepts the highest offer or negotiates with the interested buyers. Similar routines apply to many other goods and platforms. Alibaba is a perfect example. It is one of the biggest online e-commerce companies for small businesses, have hundreds of millions of users and host millions of merchants. Although information about the merchants that operate in Alibaba is not perfect, users share their experiences at Alireviews and contribute to public knowledge.

³I choose to model the competition between the buyers in a Bertrand fashion for two reasons. First, if commitments (and reputational concerns) provide some market power to the long side (i.e., the buyers) in this highly competitive situation, then they should provide even greater power when the buyers already own some due to factors other than commitments and reputation. Second, if we exclude the commitments or the obstinate types, then the model provides a clear benchmark result: the unique equilibrium outcome is the Walrasian outcome, where the monopolist seller gets the entire surplus and leaves none to the buyers.

Our main result shows that playing the tough bargainer will not help the rational players of the long side of the market (i.e., the buyers). That is, the unique equilibrium outcome is the Walrasian outcome, where the monopolist seller gets the entire rent and leaves no surplus to the buyers. The key assumption for this result is that the probability of obstinacy is constant and independent of the chosen price/posture. In a very similar model, Ozyurt (2015) shows that players' reputational concerns (even if they are negligibly small) may provide strong market power to the buyers. His result is sustained by specific "off the equilibrium path" beliefs: a deviating/overbidding buyer is believed to be an obstinate type, and being perceived as an obstinate buyer reduces the chance that his offer will be accepted by the seller because the rational seller prefers to visit first the buyer who is likely to be rational, and this restrains a rational buyer from overbidding his rival.

The analysis also provides important hints regarding how robust our main finding is and what directions one should consider. For example, a buyer's reputational advantage (against his rival or the seller) will not help him to get a positive surplus if the players' initial reputations—prior beliefs that they are obstinate types—are derived from a common prior. Assuming that both buyers' bids are the same, the seller prefers to visit the more flexible buyer (the one that is more likely to be flexible) first only when the seller is sufficiently powerful against the buyers: powerful in the sense that the seller can make a "take it or leave it" ultimatum to the first buyer he visits. If the seller is not that powerful and needs to increase the value of his outside option (by building up his reputation) before leaving the first buyer he visits, then he will be indifferent between visiting the tougher buyer and the more flexible buyer first. In general, a rational seller may prefer to visit and negotiate first with the "unattractive" buyer, who is greedier and tougher than the other buyer. This behavior is consistent with equilibrium because when the seller picks a buyer, he not only chooses his bargaining partner but also picks his outside option. If the seller picks the unattractive buyer first and leaves the attractive buyer aside, he may actually strengthen his bargaining position against both buyers because he starts the sequel with a strong outside option. Therefore, competitive outcomes (i.e., ones that are different from the monopoly pricing) would be consistent with equilibrium if the players' initial reputations are derived from a non-common priors (e.g., each buyer believes that his opponents are tougher negotiators than himself).

The key difference of the present paper with Ozyurt (2015) is that in the present paper—similar to that in Section 4 of Crawford (1982), Kambe (1999), Wolitzky (2012), and Ellingsen and Miettinen (2014)—the probability of obstinacy is independent of the chosen prices.⁴ There

⁴The game in Ozyurt (2015) is very similar to the one studied in this paper. In fact, the negotiation phase (i.e., the third stage of the game G that is defined in Section 2) of both models are identical with a qualitatively insignificant difference: the long side in Ozyurt(2015) is the sellers. The main output of Ozyurt (2015) is an existence result, whereas this paper provides a uniqueness result. Therefore, the present paper fully characterizes the equilibrium strategies of the negotiation phase, which the previous one does not. Nevertheless, the key assumptions in these two papers about the players' commitments lead to substantially different dynamics and

are two different approaches to commitments in the bargaining literature. Similar to Abreu and Gul (2000), Compte and Jehiel, (2002), and Atakan and Ekmekci (2014), Ozyurt (2015) models commitments through behavioral types: types that are born with their non-negotiable demands. Given this interpretation, if a negotiator is rational and demands a fixed surplus, then this is his strategic choice. If he is an obstinate type, then he merely declares the demand corresponding to his type. Therefore, conditional on a particular demand announcement, a posterior probability that an agent is obstinate is calculated by the rational agent's strategy and the probability distribution of the obstinate types. However, in the second category, in which the current paper falls, commitments are strategic actions that rational negotiators take by their own will. This approach is widely accepted in international relations literature.⁵ The essential idea seems to involve making a demand and "burning one's bridges," or taking actions during the negotiation process that increase the future cost of backing down from one's demand (Crawford 1982). In the same vein with Kambe (1999), Wolitzky (2012), and Ellingsen and Miettinen (2014), the present paper does not aim to explain the working principles of the mechanism that the negotiators use to "tie their hands," and thus models this mechanism as a move by nature; after they announce their prices/commitments, negotiators are forced to stick to their commitments with some probability.

The work of Atakan and Ekmekci (2014) is related to the present paper as they study a market environment with multiple buyers and sellers. However, their main focus is substantially different. In a market with large numbers of buyers and sellers, they show that the existence of obstinate types and endogenous outside options provide enough incentive for the rational players to build reputations for obstinacy.⁶ In that regard, they study a model where the sellers' and the buyers' (commitment) demands are unique and exogenously given. Moreover, the buyers and the sellers are matched randomly with some exogenous matching mechanism, and thus the negotiators cannot direct their search. Furthermore, unlike the present model, negotiators' reputations are reset when they change their partners; therefore, reputation building does not directly contribute to a negotiator's outside option.

Section 2 explains the details of the three-stage, continuous-time competitive multilateral bargaining game. Section 3 provides the main results of this paper. Section 4 characterizes the equilibrium strategies of the third stage of the game, which are essential to prove the results of Section 3. All proofs are deferred to the Appendix.

results for the initial demand announcement phase.

⁵See Ozyurt (2014) for a broader review on the crisis bargaining literature.

⁶In a sense, Atakan and Ekmekci (2014) makes a general equilibrium type of analysis of the Compte and Jehiel (2002) model by considering multiple buyers and sellers. The key difference between the two is that the negotiators' outside option is exogenous in the latter, but endogenous in the former.

2. THE COMPETITIVE (MULTILATERAL) BARGAINING GAME

Here I define the continuous-time, *competitive multilateral bargaining game* G .

The Players: There is a single seller having an indivisible good and two buyers who want to consume this good. The valuation for the good is 1 for the buyers and 0 for the seller. All three players discount time, and the rate of time preference of the buyers and the seller are r_b and r_s , respectively. All of this information is common knowledge among all three players.

The Timing of the Game: The bargaining game between the seller and the two buyers is a three-stage, infinite horizon, continuous-time game. Stage 1 starts and ends at time 0 and the timing within the first stage is as follows. Initially, the seller announces (posts) a demand (price) α_s from the set $[0, 1]$ and it is publicly observable. After observing the seller's demand, the buyers simultaneously announce their demands, α_i for buyer $i = 1, 2$, from the set $[0, 1]$. The game finishes at this point by the seller's acceptance of the highest offer if $\max\{\alpha_1, \alpha_2\} \geq \alpha_s$. In case both buyers offer α where $\alpha \geq \alpha_s$, then the seller accepts each buyer's offer with equal probabilities. However, if $\alpha_s > \max\{\alpha_1, \alpha_2\}$, then the seller selects one of the buyers to visit and to negotiate up the price.

A player knows that he will never be forced to commit to his initial demand, but is uncertain about the other players. Therefore, each player believes that nature sends one of two messages $\{c, d\}$ to his opponents in stage 2. A player who receives the message c "commit" is constrained to reject all shares that are less than what he initially claimed for himself. If a player receives the message d "don't commit", he will continue to play the game with no commitment to his initial share. The players share the same belief that the buyers and the seller receive the message c with probability z_b and z_s , respectively, where $z_b, z_s \in (0, 1)$.

Upon the beginning of the third stage (still at time 0) the seller and buyer i , who is visited by the seller first, immediately begin to play the following *concession game*: At any given time, a player either accepts his opponent's initial demand or waits for a concession. At the same time, the seller decides whether to stay or leave buyer i . If the seller leaves buyer i and goes to buyer $j \in \{1, 2\}$ with $j \neq i$, the seller and buyer j start playing the concession game upon the seller's arrival. Assuming that the buyers are spatially separated, let δ denote the discount factor for the seller that occurs due to the time $\Delta > 0$ required to travel from one buyer to the other. That is, $\delta = e^{-r_s \Delta}$. Note that $1 - \delta$ (the search friction) is the cost that the seller incurs each time he switches his bargaining partner.⁷ Concession of the seller or buyer i , while the seller is with buyer i , marks the completion of the game. If the agreement $\alpha \in \{\alpha_s, \alpha_i\}$ is

⁷It is equally acceptable to assume a switching cost for the seller that is independent of the "travel time" Δ , but this change would not affect our results. However, incorporating the search friction in this manner simplifies the notation substantially.

reached at time t , then the payoffs to the seller, buyers i and j are $\alpha e^{-r_s t}$, $(1 - \alpha)e^{-r_b t}$ and 0, respectively. In case of simultaneous concession, surplus is split equally.⁸

I denote this three-stage bargaining game by G . A critical assumption of the model deserves explicit clarification here. The model adopts a war of attrition protocol in stage 3, disallowing counteroffers and permitting buyers only two choices: concede or wait. The third stage of the game G is a modified war of attrition game and it is justified in the bargaining and reputation literature with the following arguments. Alternatively, we could suppose that players can modify their offers at times $\{1, 2, \dots\}$ in alternating orders, but can concede to an outstanding demand at any $t \in [0, \infty)$. Given the behavior of the commitment types, modifying his offer would reveal a player's flexibility, and in the unique equilibrium of the continuation game, he should concede to the opponent's demand immediately. Hence, in equilibrium, rational players would never modify their demands.⁹

The Information Structure and Some Details on Obstinate Types: The only source of uncertainty is the players' actual types, which matters only in the third stage of the game. In the first stage, all players are flexible (rational)—in the sense that they choose their strategies, given their beliefs, to maximize their expected payoffs—and this is common knowledge. Following the second stage, players are uncertain about their opponents' types. However, a player is either rational or obstinate (inflexible).

As is standard in the literature, the obstinate types follow a simple strategy: never back down from the initial offer. In particular, the obstinate type seller always demands his first stage price, α_s , accepts any price offer greater or equal to it and rejects all smaller offers. Likewise, obstinate type buyer i always demands his first stage offer α_i , accepts any price offer smaller or equal to it and rejects all greater offers. Although the remaining assumption is dispensable, I give it for the sake of completeness: I assume that the obstinate type seller understands the equilibrium and leaves his bargaining partner permanently when he is convinced that his partner will never concede. One may wish to consider the case where the obstinate seller is more strategic or aggressive in the sense that he leaves the first buyer he visits immediately in case his demand is not accepted. This assumption would certainly not alter our main result.¹⁰

Strategies of the Rational (flexible) Players: In the first stage of the bargaining game G , a strategy for the seller and buyer i is a pure action $\alpha_s, \alpha_i \in [0, 1]$. Since the subsequent analysis

⁸In this case, the seller's and buyer i 's shares are $\frac{\alpha_i + \alpha_s}{2}$ and $1 - \frac{\alpha_i + \alpha_s}{2}$, respectively. This particular assumption is not crucial because simultaneous concession occurs with probability 0 in equilibrium.

⁹See Ozyurt (2015) and also Kambe (1999), Abreu and Gul (2000), Compte and Jehiel (2002), Abreu and Pearce (2007), and Atakan and Ekmekci (2014).

¹⁰If the obstinate seller makes a "take it or leave it ultimatum" to the first buyer he visits, it would increase the flexible seller's bargaining power, not decrease it. Furthermore, as our discussions in Section 3 (in particular, Theorem 2) indicate, competitive prices would be consistent with equilibrium if the buyers are "powerless", implying that the flexible seller should be making a "take it or leave it ultimatum" to the first buyer he visits.

is quite involved and the equilibrium outcome is unique, I will restrict my attention to pure strategies in the demand announcement phase of the first stage. Let σ_i denote the probability that the rational seller visits buyer i first, and so $\sigma_1 + \sigma_2 = 1$. Although the buyers' strategies α_1, α_2 are functions of the seller's announcement α_s , and σ_i is a function of all three players' announcements, these connections are omitted for notational simplicity. The probabilities that the seller and the buyers are obstinate are z_s and z_b , respectively, and this is true independent of the players' strategies in the first two stages.

Third-stage strategies are relatively more complicated. A nonterminal history of length t , h_t , summarizes the initial demands chosen by the players in stage 1, the sequence of buyers the seller visits and the duration of each visit until time t (inclusive). For each $i = 1, 2$, let \hat{H}_t^i be the set of all nonterminal histories of length t such that the seller is with buyer i at time t . Also, let H_t^i denote the set of all nonterminal histories of length t with which the seller arrives at buyer i at time t .¹¹ Finally, set $\hat{H}^i = \bigcup_{t \geq 0} \hat{H}_t^i$ and $H^i = \bigcup_{t \geq 0} H_t^i$.

The seller's strategy in the second stage has three parts. The first part determines the seller's location at any given history. For the other two parts, \mathcal{F}_s^i for each i , let \mathbb{I} be the set of all intervals of the form $[T, \infty] \equiv [T, \infty) \cup \{\infty\}$ for $T \in \mathbb{R}_+$, and \mathbb{F} be the set of all right-continuous distribution functions defined over an interval in \mathbb{I} . Therefore, $\mathcal{F}_s^i : H^i \rightarrow \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F_s^{i,T} : [T, \infty] \rightarrow [0, 1]$ representing the probability of the seller conceding to buyer i by time t (inclusive). Similarly, buyer i 's strategy $\mathcal{F}_i : H^i \rightarrow \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F_i^T : [T, \infty] \rightarrow [0, 1]$ representing the probability of buyer i conceding to the seller by time t (inclusive).

Reputation of player $n \in \{1, 2, s\}$ (i.e., \hat{z}_n), representing the probability that the other players attach to the event that player n is the obstinate type, is a function of player n 's strategy and past history. It is updated according to Bayes' rule. For example, given a history h_0 where the seller announces α_s and visits buyer i first, the seller's reputation at the time he visits buyer i (i.e., $\hat{z}_s(h_0)$) is z_s . Following the history h_0 , if the seller plays the concession game with buyer i until some time $t > 0$, and the game has not ended yet (call this history h_t), then the seller's reputation at time t is $\frac{\hat{z}_s(h_0)}{1 - F_s^{i,0}(t)}$, assuming that the seller's strategy in that concession game against buyer i is $F_s^{i,0}$.

Note from the last arguments that the seller's reputation at time t reaches 1 when $F_s^{i,0}(t)$ reaches $1 - \hat{z}_s(h_0)$. This is the case because $F_s^{i,0}(t)$ is the buyers' belief about the seller's play during the concession game with buyer i . That is, *it is the strategy of the seller from the point of view of the buyers*. More generally, the upper limit of the distribution function $F_s^{i,T}$ is $1 - \hat{z}_s(h_T)$ where $\hat{z}_s(h_T)$ is the seller's reputation at time $T \geq 0$, the time that the seller (re)visits buyer i . The same arguments apply to the buyers' strategies.

¹¹That is, there exists $\epsilon > 0$ such that for all $t' \in [t - \epsilon, t)$, $h_{t'} \notin \hat{H}_t^i$ but $h_t \in \hat{H}_t^i$.

Given F_i^T , the rational seller's expected payoff of waiting until time t and conceding to buyer i at this time is

$$U_s^i(t, F_i^T) \equiv \alpha_s \int_0^{t-T} e^{-r_s y} dF_i^T(y) + \frac{1}{2}(\alpha_i + \alpha_s)[F_i^T(t) - F_i^T(t^-)]e^{-r_s(t-T)} + \alpha_i[1 - F_i^T(t)]e^{-r_s(t-T)} \quad (1)$$

with $F_i^T(t^-) = \lim_{y \uparrow t} F_i^T(y)$. The first term of the payoff function in (1) is the seller's (expected) payoff conditional on the event that the game ends before time t with buyer i 's concession. The last term is the seller's payoff conditional on the event that buyer i does not concede before time t , and the term in the middle is conditional on buyer i conceding at exactly time t .

In a similar manner, given $F_s^{i,T}$, the expected payoff of rational buyer i who waits until time t and concedes to the seller at this time is

$$U_i(t, F_s^{i,T}) \equiv (1 - \alpha_i) \int_0^{t-T} e^{-r_b y} dF_s^{i,T}(y) + \frac{1}{2}(2 - \alpha_i - \alpha_s)[F_s^{i,T}(t) - F_s^{i,T}(t^-)]e^{-r_b(t-T)} + (1 - \alpha_s)[1 - F_s^{i,T}(t)]e^{-r_b(t-T)} \quad (2)$$

where $F_s^{i,T}(t^-) = \lim_{y \uparrow t} F_s^{i,T}(y)$. The first term of the payoff function in (2) indicates buyer i 's payoff conditional on the event that the game ends before time t with the seller's concession. The last term is buyer i 's payoff conditional on the seller not conceding to buyer i before time t . Both (1) and (2) are evaluated at time T , and are conditional on the event that the seller visits buyer i at time $T \geq 0$.

3. THE MAIN RESULTS

I will start by presenting the first result that characterizes the equilibrium price selections of the players in the first stage of the bargaining game G. The main message is that the unique equilibrium outcome is the Walrasian outcome, where the monopolist seller gets the entire rent and leaves no surplus to the buyers. The proof does not immediately follow from standard Bertrand-like price competition models. After presenting the result, I give a detailed summary of the equilibrium and a sketch of its proof. The complete characterization of the equilibrium strategies of the third stage of the game G is presented in Section 4 and all proofs are deferred to the Appendix.

Theorem 1. *There always exists a sequential equilibrium of the game G, in which the seller posts the price of 1 and both buyers announce 1 regardless of the seller's announcement, and thus, the game ends at time 0. Moreover, the monopoly price is the unique equilibrium outcome of the game G. That is, there does not exist a sequential equilibrium where the players' realized price announcements in stage 1 satisfy $1 \geq \alpha_s \geq \alpha_1, \alpha_2$ such that at least one of the inequalities is strict.*

I am particularly interested in equilibrium outcomes of the game G, where the market frictions (i.e., the players' initial reputations and the search cost) are small. However, Theorem 1 holds regardless of the size of these frictions. The existence part of this result is simple: if buyer 2 announces his demand as 1, then there is no profitable deviation for buyer 1. The uniqueness of the equilibrium outcome is not straightforward. The reason is that overbidding the opponent is not always an optimal deviation strategy for the buyers because higher price does not necessarily attract the seller. I will elaborate more on this point at the end of this section.

It is also easy to see that there is no equilibrium in which $1 > \alpha_s$ and $\alpha_i = \alpha_s$ for some $i \in \{1, 2\}$. If there were such an equilibrium, then when $\alpha_1 = \alpha_s > \alpha_2$, the game would end at time 0 with the seller's acceptance of α_1 , and thus, buyer 2's payoff would be 0. However, buyer 2 would profitably deviate to a price $\alpha_s + \epsilon$, where $\epsilon > 0$ is small, contradicting the optimality of equilibrium. The case where $\alpha_s = 1 > \alpha_1, \alpha_2$ is deferred to the Appendix. The intriguing part is to prove that there exists no equilibrium in which the players' price announcements satisfy $1 > \alpha_s > \alpha_1, \alpha_2$. Before I sketch its proof, I would like to give a short descriptive summary of the equilibrium of the third stage of the game G. For this purpose, it may be beneficial to consider the following benchmark result, following from Abreu and Gul (2000).

A Simple Benchmark Result: Suppose for now that there is only one buyer, denoted by b , with a unique demand to announce $\alpha_b \in (0, 1)$, which is incompatible with the seller's demand $\alpha_s \in (0, 1)$. The timing of the modified version of the game G goes as follows. In stage 1, the seller and then the buyer announce their demands. Since each player has a unique demand, this stage has no strategic content. In the third stage (still at time 0), players begin to play the concession game as described in Section 2 with one important difference: the seller has no outside option of leaving the buyer. This version of the model is identical to the single-type setup of Abreu and Gul (2000), and the *unique* equilibrium strategies are characterized by the following three conditions:

$$\begin{aligned} F_n(t) &= 1 - c_n e^{-\lambda_n t} \text{ for all } t \leq T^e \\ c_n &\in [0, 1], (1 - c_b)(1 - c_s) = 0, \text{ and} \\ F_n(T^e) &= 1 - z_n \text{ for all } n \in \{b, s\} \end{aligned} \tag{3}$$

where $\lambda_b = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha_b}$ and $\lambda_s = \frac{\alpha_b r_s}{\alpha_s-\alpha_b}$. During the concession game, the flexible buyer and seller concede by choosing the timing of acceptance randomly with constant hazard rates λ_b and λ_s , respectively. They play the concession game until time T^e , at which point both players' reputations simultaneously reach 1. Since rational player n is indifferent between conceding and waiting at all times, his expected payoff during the concession game (i.e., v_n) is equal to what

he can achieve at time 0. Therefore, by Equations (??) and (??) we have

$$\begin{aligned} v_b &= F_s(0)(1 - \alpha_b) + [1 - F_s(0)](1 - \alpha_s), \text{ and} \\ v_s &= F_b(0)\alpha_s + [1 - F_b(0)]\alpha_b \end{aligned} \quad (4)$$

Note that $(1 - c_n)$ indicates the probability of player n 's initial concession, and the second condition of (??) (i.e., $(1 - c_b)(1 - c_n) = 0$) implies that only one player can make concession at time 0. This is a standard result in continuous-time concession games; if a player immediately concedes with a positive probability, then his opponent prefers to wait in order to enjoy the discrete chance of concession. In equilibrium, Abreu and Gul (2000) call a player **strong** if his opponent makes an initial probabilistic concession at time 0 and **weak** otherwise. If one solves the three equalities in (??) for the unknowns c_s, c_b and T^e , we find that the seller is strong if and only if $z_s > z_b^{\lambda_s/\lambda_b}$.¹² Moreover, the second condition of (??) implies that if the buyer is strong, then the seller must be weak (or, conversely, if the seller is strong, then the buyer must be weak). In fact, both players can be weak, but both of them cannot be strong. Finally, equations in (??) imply that the equilibrium payoffs of the flexible buyer and the seller when they are weak are $(1 - \alpha_s)$ and α_b , respectively.

The Third Stage of the Game G: Now, I resume the analysis of the game G. Equilibrium strategies of the third stage of the game depend on the demands declared in stage 1. Suppose that the buyers' announcements are α_1 and α_2 , and that $\alpha_s > \alpha_1 \geq \alpha_2$. There are two main cases to consider. The first case is $\delta\alpha_1 > \alpha_2$. That is, the buyers' posted prices are significantly apart from each other. In this case, the seller never plays the concession game with the greedy buyer (i.e., buyer 2) because accepting buyer 1's demand is strictly better than conceding to buyer 2. The buyers' distinct demands give the seller strong incentive to make a "take it or leave it" ultimatum to buyer 2. If the seller ever visits buyer 2 and if his ultimatum is not accepted, then the seller immediately leaves buyer 2 and goes to buyer 1. The seller and buyer 1 concede by choosing the timing of acceptance randomly with constant hazard rates, which are slightly modified versions of the ones defined above (i.e., λ_s and λ_b). Depending on the parameters, the seller may visit buyer 2 first and buyer 1 afterwards.

The second case is $\alpha_2 \geq \delta\alpha_1$. That is, the buyers' posted prices are close to each other. If z_s is sufficiently small relative to z_b , then the seller's initial bargaining power will be weak, so that leaving a buyer will never be an optimal action unless the seller builds his reputation for obstinacy. The seller will visit each buyer with a positive probability. If the seller first visits buyer 1, for example, then he plays the concession game with this buyer until some deterministic time $T_1^d \geq 0$, which depends on the primitives and the first-stage prices. Unless buyer 1 or the seller concedes prior to time T_1^d , the seller leaves buyer 1 at this time and immediately goes to

¹²Therefore, the seller is weak if and only if the inverse of this inequality holds.

buyer 2 to play the concession game with him. Since the equilibrium in the concession games are in mixed strategies, the seller builds up his reputation for obstinacy while negotiating with the buyers. Thus, the seller’s reputation for obstinacy will be higher when the seller leaves buyer 1. The seller visits each buyer at most once, and so, a rational buyer will never allow the seller to leave him without reaching a deal. Put it differently, when the seller leaves a buyer, he will be convinced that this buyer is obstinate.

In equilibrium, the flexible seller plays the concession game with buyer i for a while if the seller (1) is indifferent between, on the one hand, accepting buyer i ’s demand, thus receiving the instantaneous payoff of α_i , and on the other hand, waiting for the concession of buyer i , and (2) prefers accepting buyer i ’s demand over his endogenous outside option: visiting the other buyer and playing the concession game with that one. When the seller picks which buyer to visit first, he not only chooses his bargaining partner but also picks his outside option. If the seller picks the “strong” buyer first and leaves the “weak” buyer aside, he may actually strengthen his bargaining position against both buyers because he starts the sequel with a strong outside option. The strength of a player will be determined in equilibrium, but unlike Abreu and Gul (2000), a buyer’s strength in our case depends not only on his and the seller’s prices and initial reputations, but also on the seller’s outside option (i.e., the other buyer’s price and reputation).

Definition 1. *Buyer i is called **strong** if the rational seller concedes to buyer i with a positive probability at the time the seller visits buyer i first at time 0 and **weak** otherwise. Similarly, the seller is called **strong against buyer i** if rational buyer i concedes to the seller with a positive probability at the time the seller visits buyer i first at time 0 and **weak against buyer i** otherwise.*

This definition is in line with the definition of weak (or strong) player of Abreu and Gul (2000). However, what it implies—in terms of the relationship between prices and initial priors—is very different. According to the equilibrium strategies of the third stage of the game G, if $\delta\alpha_1 > \alpha_2$, then buyer 2 is weak regardless of the seller’s price α_s and the initial priors z_b and z_s (Proposition 4). On the other hand, buyer 1’s strength depends on all these variables in a rather complicated way (see Propositions 5-7). If the buyers’ posted prices are close to each other (i.e., $\alpha_2 \geq \delta\alpha_1$), then the seller is weak against, for example, buyer 2 if and only if

$$(z_b/A_2)^{\lambda_s^1/\lambda_1} z_b^{\lambda_s^2/\lambda_2} \geq z_s \quad (5)$$

assuming that $\lambda_s^i/\lambda_i = \frac{(1-\alpha_s)r_b}{\alpha_i r_s}$ and $A_2 = \frac{\delta\alpha_s - \alpha_2}{\delta(\alpha_s - \alpha_1)} > z_b$.¹³ A complete characterization of the strong (or the weak) player can be derived by Remarks 2-8 in Section 4.

¹³Therefore, the seller is strong against buyer 2 (or buyer 2 is weak) if and only if $z_s > (z_b/A_2)^{\lambda_s^1/\lambda_1} z_b^{\lambda_s^2/\lambda_2}$ assuming that λ_s^i/λ_i and A_2 satisfy the above conditions. Note that the first and the second z_b in inequality (??) indicate the initial reputations of buyer 1 and 2, respectively. That is, if the buyers’ initial reputations would have been $z_1 \neq z_2$, then the inequality (??) should be rewritten as $(z_1/A_2)^{\lambda_s^1/\lambda_1} z_2^{\lambda_s^2/\lambda_2} \geq z_s$.

Properties of the Equilibrium of the Game G: In any sequential equilibrium of the game G, following a history where the players' prices satisfy $1 > \alpha_s > \alpha_1 \geq \alpha_2$, the following arguments are true:

1. If buyer i is strong, then the seller must be weak against buyer i . If the seller is strong against buyer i , then buyer i must be weak. All three players can be weak, but all of them cannot be strong: *As discussed above, this is a well established result in continuous-time war of attrition (or concession) games.*
2. If buyer i is weak, then his expected payoff in the game is $\sigma_i(1 - \alpha_s)$ and σ_i , representing the probability that the seller visits buyer i first, is equal to 1: *The flexible buyers' concession game payoffs are calculated as in equations (??). If $\sigma_i < 1$ or buyer i 's payoff is $1 - \alpha_s - \epsilon$ for some $\epsilon > 0$, then buyer i can profitably deviate to the price $\alpha_s + \epsilon/2$.*
3. If the seller is weak against both buyers, then (1) the buyers' prices must satisfy $\alpha_2 > \delta\alpha_1$, (2) the seller's continuation payoff visiting buyer i first is α_i , and (3) the seller visits buyer 1 first: *As mentioned above, if the buyers' prices are significantly apart from each other (i.e., $\delta\alpha_1 \geq \alpha_2$), then the seller is strong against buyer 2. Definition 1 implies the second part because the flexible seller's concession game payoffs are calculated as in equations (??). The optimality of equilibrium automatically implies the third part.*
4. If the buyers' prices are not so far apart from each other (i.e., $\alpha_2 > \delta\alpha_1$), then the game payoff of the buyer who is visited in second place is strictly less than $(1 - \alpha_s)$: *Recall that when $\alpha_2 > \delta\alpha_1$ holds, the seller's outside option is low enough to make a credible "take it or leave it offer," and the seller does not visit a buyer more than once. Thus, the flexible seller cannot leave the first buyer he visits unless his reputation grows high enough against the second buyer, in which case the second buyer concedes to the seller with a positive probability at the beginning of the concession game between the seller and the second buyer. Because the second buyer is the player who concedes at the beginning of the (last) concession game, the benchmark results and the equations (??) imply that the second buyer's continuation payoff must be $(1 - \alpha_s)$.¹⁴ Since the seller visits the second buyer only if the first one is the commitment type, then the second buyer's game payoff is at most $z_b(1 - \alpha_s)$.*
5. If the buyers' prices are significantly apart from each other (i.e., $\delta\alpha_1 > \alpha_2$), then flexible buyer 2 accepts the seller's price (regardless of its size) immediately. Thus, buyer 2 is weak regardless of the priors: *I already discussed this point above.*

¹⁴That is, buyer i 's continuation game payoff following a history where the seller visits buyer i after he negotiates with buyer j first must be $(1 - \alpha_s)$.

A Sketch of the Proof of Theorem 1: Now, suppose for a contradiction that there exists an equilibrium in which the players' price announcements in stage 1 satisfy $1 > \alpha_s > \alpha_1, \alpha_2$. I reach a contradiction in four main steps. The first step shows that at least one of the buyers must be weak. The second step shows that if one buyer is weak, then the buyers' announcements must be different. The third step shows that the first buyer (who makes the highest bid) cannot be the weak buyer. As a result of these three steps, we can conclude that in equilibrium it must be true that $1 > \alpha_s > \alpha_1 > \alpha_2$, buyer 1 is strong, and 2 is weak. However, the fourth step shows that there is no such equilibrium.

In order to prove the first step (i.e., at least one buyer must be weak) I suppose, for a contradiction, that both buyers are strong. This claim implies that (with the first property above) the seller must be weak against both buyers, and thus the seller strictly prefers to visit the buyer who posts the higher price first (property 3). As a result of this, each buyer will have an incentive to overbid his opponent, and so the buyers would deviate unless α_1 and α_2 are equal to 1, contradicting our starting assumption that $1 > \alpha_s > \alpha_1, \alpha_2$. Hence, we can conclude that at least one buyer must be weak in equilibrium.

To prove the second step, I suppose that one buyer is weak, and for a contradiction, that the buyers' announcements are equal. Then both buyers must be weak because the buyers are identical (i.e., the buyers' initial reputation z_b , time preference r_b , and prices are the same) and the players' strength depends solely on these parameters.¹⁵ However, by property 2, the seller must visit both buyers in equilibrium with probability 1, which is not possible.

For the third step, I assume for a contradiction that the realized announcements satisfy $1 > \alpha_s > \alpha_1 > \alpha_2$ and buyer 1 is weak. Then, by property 2 we know that the seller shall visit buyer 1 first with probability 1. Moreover, properties 4 and 5 imply that the second buyer's expected payoff in the game is strictly less than $1 - \alpha_s$ (as the seller will visit buyer 2 after visiting buyer 1). However, 2 can profitably deviate by posting a price $\alpha_s + \epsilon$, where $\epsilon \geq 0$ is sufficiently small, contradicting the optimality of the equilibrium.

The last step is trickier. I suppose for a contradiction that the players' prices satisfy $1 > \alpha_s > \alpha_1 > \alpha_2$ and that buyer 2 is weak. Property 2 implies that the seller shall visit buyer 2 first and buyer 2's payoff in the game is $1 - \alpha_s$. If the buyers' prices are close to each other (i.e., $\alpha_2 > \delta\alpha_1$), then buyer 1's game payoff will be strictly less than $1 - \alpha_s$ (property 4), implying that buyer 1 can profitably deviate to the price α_s . On the other hand, if the buyers' prices are apart from each other, then I consider two exhaustive subcases and reach the desired contradiction in each one of them. First, if $z_s \geq z_b^{\lambda_s/\lambda_1}$ holds, then with a reasoning similar to Property 4, buyer 1's game payoff is strictly less than $1 - \alpha_s$, and so buyer 1 would profitably deviate to α_s . Second, if $z_b^{\lambda_s/\lambda_1} > z_s$ holds, then buyer 2 can deviate to a price very close to α_s and ensure that

¹⁵One can prove the second step without using the fact that the buyers are identical. However, using the buyers' similarity is an effective shortcut.

he is strong and that the seller will visit him first because the buyer is offering almost what the seller wants, implying a game payoff slightly (but strictly) higher than $1 - \alpha_s$.

Further Discussions about the Equilibrium

A closer look at the players' "off the equilibrium path" behaviors is necessary to understand how robust Theorem 1 is, and this is what I do in this section. As the arguments in the previous section highlight, two items appear to be important for Theorem 1. The first one is the buyers' incentives to get the seller first and the second one is the buyers' homogeneity. Being the seller's first host (i.e., bargaining partner) is crucial for the buyers because the rational buyer who is visited by the seller first makes the agreement with the seller. We know this from the equilibrium strategies of the third stage of the game G, which are presented in Section 4. However, higher prices and higher or lower initial reputation do not always guarantee the seller's first visit.

Assume that the buyers' initial reputations in the bargaining game G satisfy $1 > z_1 > z_2 > 0$. That is, buyer 1 is believed to be tougher than buyer 2 (i.e., buyer 1 is more likely to be the obstinate type). The next result shows that being the more flexible buyer may be beneficial for buyer 2, but not always. Define $z_s^* \equiv \left(\frac{z_2}{A}\right)^{\lambda_s/\lambda_b}$ where $A = \frac{\delta\alpha_s - \alpha_b}{\delta(\alpha_s - \alpha_b)}$ and $\lambda_s/\lambda_b = \frac{(1-\alpha_s)r_b}{\alpha_b r_s}$. Let h^* denote a history of the game G where the seller posts α_s and both buyers post α_b in the first stage such that $1 > \alpha_s > \alpha_b > 0$ and $A > z_1$ (i.e., the seller's price is not too close to the buyers' price).

Theorem 2. *Assume that the buyers' initial reputations in the bargaining game G satisfy $1 > z_1 > z_2 > 0$. In any sequential equilibrium of the game G, following the history h^* , the seller prefers to visit buyer 2 first if and only if $z_s > z_s^*$. For all other values of z_s , the seller is indifferent between the buyers.*

Some implications of the last result are worth mentioning. The threshold z_s^* is critical because when z_s is larger than this threshold, the seller is strong against both buyers, and his outside option at time 0 is so powerful that he can make a "take it or leave it" ultimatum to the tougher buyer (i.e., buyer 1) when the seller first visits him. However, when z_s is smaller than this threshold, the seller may or may not be strong against the buyers (depending on how close z_s is to 0), but his outside option at time 0 is no longer powerful enough to make a "take it or leave it" ultimatum to any buyer. In fact, if $z_s^* > z_s$, then the seller has to build up his reputation and increase the value of his outside option before leaving the first buyer he visits.

Conventional wisdom suggests that the seller should pick the tougher buyer when he is "powerful" enough (in the sense that the seller can make a "take it or leave it" ultimatum) against this buyer. However, Theorem 2 suggests, rather unconventionally, that the seller prefers to stay away from the tougher buyer when the seller is powerful against him, but the seller is

reluctant to pick the tougher buyer when the seller is not powerful against him. This behavior is consistent with equilibrium because leaving the tougher buyer aside when the seller is stronger creates a valuable outside option for the seller that strengthens the seller’s overall bargaining power. The next example may reveal this intuition more clearly.

Consider the equilibrium of the game G , following a history where the players’ prices satisfy $1 > \alpha_s > \alpha_2 > \alpha_1$. That is, buyer 1 is greedier and tougher (because $z_1 > z_2$). The seller prefers—for some parameter values—to visit buyer 1 first even though buyer 1 is less “attractive.” This behavior occurs when the buyers’ prices are significantly apart from one another (i.e., $\delta\alpha_2 > \alpha_1$) and the seller’s reputation is sufficiently low, in which case the flexible seller can never leave buyer 2 (if he ever visits buyer 2 first), whereas he can make a “take it or leave it” ultimatum to buyer 1 (if he visits buyer 1 first). The seller picks seemingly unattractive buyer first because his outside option when he visits this buyer is much more valuable than his outside option when he first visits the attractive buyer (i.e., buyer 2).

The second item that appears to be important for Theorem 1 is the buyers’ homogeneity. As I will briefly discuss it next, it is not a critical force behind Theorem 1, and so differentiation of the buyers regarding their flexibility will not soften the competition. I still assume that the buyers’ initial reputations in the bargaining game G satisfy $1 > z_1 > z_2 > 0$. Suppose, for a contradiction, that there is an equilibrium of the game G where $1 > \alpha_s > \alpha_1, \alpha_2$. I will confine my attention to particular cases because the complete proof of Theorem 1 under the new scenario would be much lengthier than the current version. In an extreme case where $z_2 = 0$ (or very close to 0), the one-sided reputation result of Myerson (1991) implies that buyer 2 immediately accepts any price the seller posts. The seller will visit buyer 2 first because it is a sure lottery with the prize α_s . Therefore, buyer 1’s game payoff is 0. However, buyer 1 can profitably deviate to a price $\alpha_s + \epsilon$ for a small $\epsilon > 0$, contradicting the optimality of equilibrium.

Suppose now that the values of z_1 and z_2 are small, but z_2 is not too small as in the previous case. First of all, at least one buyer must be weak. The idea of this is exactly the same as the first step of the proof of Theorem 1, which is summarized above. The weak buyer—let us assume, without loss of generality, that it is buyer 1—must be visited by the seller first with certainty (same as property 2). Therefore, buyer 2 will be visited by the seller in the second place, and thus, his game payoff will be at most z_1 ; the seller leaves buyer 1 only if buyer 1 is the commitment type, which is the case with probability z_1 . However, buyer 2 can deviate to α_s and ensure the payoff of $(1 - \alpha_s)$. If z_1 is small enough, then this deviation is profitable. On the other hand, if z_1 is not so small (i.e., α_s is much closer to 1 than z_1 is to 0), then buyer 1 can make himself strong by posting a price $\alpha_s - \epsilon$ for some $\epsilon > 0$ small enough¹⁶ and increase his payoff slightly over $(1 - \alpha_s)$, contradicting the optimality of equilibrium. As a result, the

¹⁶This is true because buyer 1’s strength depends on the relationship between $z_1^{\lambda_s^1/\lambda_1}$ and z_s , and the former term reaches very high values as α_s approaches 1.

monopoly price will be the unique equilibrium outcome for small values of z_1 and z_2 . In fact, similar arguments will lead to the same conclusion when z_1 and z_2 take large values.

Therefore, the buyers' heterogeneity about their flexibility does not weaken the intensity of the competition between the buyers. However, allowing non-common priors and assuming that each buyer is overconfident about his flexibility would soften the competition. One example of the buyers' overconfidence is that each buyer believes that he is more flexible than his opponents, and so, each buyer has a different model of the world that is inconsistent with a common prior. One may consider the homogeneous-product Bertrand duopoly model (a single buyer and two sellers case) to conceive why heterogeneous priors would soften competition so much. In the bare-bones version of that model, the buyer wants exactly one unit, valued at v . The two identical sellers who have an opportunity cost of $c < v$ simultaneously set prices, and the buyer purchases at the lowest price. The unique equilibrium price equals the sellers' cost. Suppose now instead that each seller believes that the buyer "prefers trading with me". Specifically, let each seller believe that the buyer values "my product" at v and "the opponent's product" at $v - \epsilon$, where ϵ is small compared to v . In this case, although the priors can be very close to the truth (when ϵ is close to 0), the game has an equilibrium in which both buyers set the monopoly price v . The reason is that both buyers are convinced that the seller will buy from them rather than from the competitor if the two prices are the same.

4. EQUILIBRIUM STRATEGIES OF THE THIRD STAGE OF THE GAME G

In this section I will characterize the rational (flexible) players' equilibrium strategies in the third stage of the game G. For each $i \in \{1, 2\}$ and $\alpha_s, \alpha_i \in [0, 1]$ where $1 > \alpha_s > \alpha_i > 0$, define the coefficients (i.e., the hazard rates) λ_s^i and λ_i as follows

$$\lambda_s^i = \frac{(1 - \alpha_s)r_b}{\alpha_s - \alpha_i} \quad \text{and} \quad \lambda_i = \frac{\alpha_i r_s}{\alpha_s - \alpha_i}$$

Although λ_s^i and λ_i depend on the players first stage choices (i.e., α_s and α_i), this connection is omitted for notational simplicity.

Proposition 1. *In any sequential equilibrium of the bargaining game G following a history h_T , where players' price announcements (bids) are $1 > \alpha_s > \alpha_1, \alpha_2$, the seller arrives at buyer i at time T and his actual type has not yet been revealed, the players' concession game strategies are $F_s^{i,T}(t) = 1 - c_s^i e^{-\lambda_s^i(t-T)}$ and $F_i^T(t) = 1 - c_i e^{-\lambda_i(t-T)}$ for all $t \geq T$, where $c_s^i, c_i \in [0, 1]$ and $F_s^{i,T}(T)F_i^T(T) = 0$.*

4.1 The Gap Between the Buyers' Bids is Small (i.e., $\alpha_2 > \delta\alpha_1$)

The results in this subsection characterize equilibrium strategies of the players following a history where the seller visits buyer 2 first and the players' prices in the first stage satisfy $1 > \alpha_s > \alpha_1 \geq \alpha_2 > 0$ and $\delta\alpha_1 \geq \alpha_2$. One can easily find the equilibrium strategies following a history where the seller visits buyer 1 first by interchanging the numbers 1 and 2 in the subscripts and superscripts of the following strategies and inequalities that involve z_b or z_s . Note the following inequality:¹⁷

$$A_2 \equiv \frac{\delta\alpha_s - \alpha_2}{\delta(\alpha_s - \alpha_1)} > z_b \quad (6)$$

Remark 1. The next result proves that the seller will not visit a buyer twice in equilibrium whenever $\alpha_2 > \delta\alpha_1$ holds. Therefore, I will use F_s^i and F_i for each i to indicate the players' third stage strategies. Although these strategies depend on the history of the game, I omit this connection for notational simplicity. Furthermore, I will manipulate the subsequent notation and reset the clock once the seller and a buyer begins a concession game. Thus, I define each player's concession game strategies (distribution functions) as if the concession game with each buyer starts at time 0.

Proposition 2. *In any sequential equilibrium where the players' initial bids satisfy the inequalities (??) and $\alpha_2 > \delta\alpha_1$, the rational seller visits each buyer at most once and a rational buyer does not allow the seller leave him without reaching an agreement. Moreover, the players' concession game strategies must satisfy*

$$\begin{aligned} F_s^2(t) &= 1 - c_s^2 e^{-\lambda_s^2 t} & F_2(t) &= 1 - z_b e^{\lambda_2(T_2^d - t)} \\ F_s^1(t) &= 1 - e^{-\lambda_s^1 t} & F_1(t) &= 1 - z_b e^{\lambda_1(T_1^e - t)} \end{aligned}$$

where

$$F_s^2(0)F_2(0) = 0 \quad \text{and} \quad F_s^1(T_1^e) = 1 - \frac{z_s}{1 - F_s^2(T_2^d)}$$

given that the rational seller visits buyer 2 first, leaves 2 at time T_2^d and no player concedes beyond time T_1^e .

Next results characterize the times that the concession games with buyer 1 and 2 ends (i.e., T_1^e and T_2^d , respectively), and the rational seller's initial concession probability (i.e., $F_s^2(0)$). The rational players' equilibrium payoffs in the concession games are calculated by the equations at (??). That is, for each buyer i

$$\begin{aligned} v_s^i &= F_i(0)\alpha_s + [1 - F_i(0)]\alpha_i, \text{ and} \\ v_i &= F_s^i(0)(1 - \alpha_i) + [1 - F_s^i(0)](1 - \alpha_s) \end{aligned} \quad (7)$$

In equilibrium, where the seller first visits buyer 2, the rational seller leaves the buyer when

¹⁷For the equilibrium strategies following a history where the seller visits buyer 1 first, one needs to consider the inequality $A_1 \equiv \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \alpha_2)} > z_b$ in what follows.

he is convinced that this buyer is obstinate. At this moment, abandoning buyer 2 is optimal for the rational seller if his discounted continuation payoff of negotiating with buyer 1 (i.e., δv_s^1) is no less than α_2 , payoff to the rational seller if he concedes to buyer 2. Let z_s^* denote the level of reputation required to provide the rational seller enough incentive to leave buyer 2. Assuming that $z_s < z_s^*$ (i.e., the rational seller needs to build up his reputation before walking out of negotiation with buyer 2), the game ends with buyer 1 at time $T_1^e = -\log(z_s^*)/\lambda_s^1$.¹⁸ Thus, z_s^* must solve $\alpha_2 = \delta v_s^1$, and given the value of $F_1(0)$ by Proposition 2, we must have $\alpha_2 = \delta \left[\alpha_1 + (\alpha_s - \alpha_1)(1 - z_b(z_s^*)^{-\lambda_1/\lambda_s^1}) \right]$ implying that $z_s^* = \left(\frac{z_b}{A_2} \right)^{\lambda_s^1/\lambda_1}$ and $A_2 = \frac{\delta \alpha_s - \alpha_2}{\delta(\alpha_s - \alpha_1)}$. Note that z_s^* is well-defined (i.e., $z_s^* \in (0, 1)$) because A_2 is positive.

Lemma 1. *Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 2 first and the players' initial bids satisfy the inequalities (??), $\alpha_2 > \delta \alpha_1$, and $z_s \geq z_s^* = (z_b/A_2)^{\lambda_s^1/\lambda_1}$. The continuation strategies will be as follows. The rational seller makes a “take it or leave it” offer to buyer 2 and goes directly thereafter to buyer 1, if the offer to buyer 2 is not accepted. Rational buyer 2 immediately accepts the seller's demand and finishes the game at time 0 with probability 1. In case buyer 2 does not concede to the seller, the seller infers that 2 is obstinate, and so he never comes back to negotiate with this buyer again. The concession game with the first buyer may continue until the time $T_1^e = -\log z_s/\lambda_s^1$ with the following strategies: $F_s^1(t) = 1 - e^{-\lambda_s^1 t}$ and $F_1(t) = 1 - z_b z_s^{-\lambda_1/\lambda_s^1} e^{-\lambda_1 t}$.*

Remark 2. If $z_s \geq (z_b/A_2)^{\lambda_s^1/\lambda_1}$ holds, then the seller is strong against buyer 2 and his equilibrium payoff of visiting buyer 2 first is given by Equations (??) as follows:

$$V_s^2 = (1 - z_b)(\alpha_s) + \delta z_b [v_s^1] = \alpha_s \left[1 - z_b(1 - \delta) - \frac{\delta z_b^2}{z_s^{\lambda_1/\lambda_s^1}} \right] + \alpha_1 \frac{\delta z_b^2}{z_s^{\lambda_1/\lambda_s^1}} \quad (8)$$

Lemma 2. *Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 2 first and the players' initial bids satisfy the inequalities (??), $\alpha_2 > \delta \alpha_1$, and $(z_b/A_2)^{\lambda_s^1/\lambda_1} z_b^{\lambda_s^2/\lambda_2} \geq z_s$. The continuation strategies will be as follows. The seller leaves buyer 2 at time $T_2^d = -\log(z_b)/\lambda_2$ for sure, if the game has not yet ended, and goes directly to buyer 1. The concession game with buyer 1 may continue until the time $T_1^e = -\log(z_b/A_2)/\lambda_1$. The players' concession game strategies are $F_s^2(t) = 1 - z_s(z_b/A_2)^{-\lambda_s^1/\lambda_1} z_b^{-\lambda_s^2/\lambda_2} e^{-\lambda_s^2 t}$, $F_2(t) = 1 - e^{-\lambda_2 t}$, $F_s^1(t) = 1 - e^{-\lambda_s^1 t}$ and $F_1(t) = 1 - A_2 e^{-\lambda_1 t}$.*

Remark 3. If $(z_b/A_2)^{\lambda_s^1/\lambda_1} z_b^{\lambda_s^2/\lambda_2} \geq z_s$ holds, then the seller is weak against buyer 2 and his equilibrium payoff of visiting buyer 2 first V_s^2 is α_2 .

Lemma 3. *Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 2 first and the players' initial bids satisfy the inequalities (??), $\alpha_2 > \delta \alpha_1$, and $(z_b/A_2)^{\lambda_s^1/\lambda_1} > z_s > (z_b/A_2)^{\lambda_s^1/\lambda_1} z_b^{\lambda_s^2/\lambda_2}$. The continuation strategies will be as follows. The seller*

¹⁸According to Proposition 1 we have $F_s^1(T_1^e) = 1 - z_s^*$, which implies the value of T_1^e .

leaves buyer 2 at time $T_2^d = -\log(z_s)/\lambda_s^2 + (\lambda_s^1/\lambda_1) \log(z_b/A_2)/\lambda_s^2$ for sure, if the game has not yet ended, and goes directly to buyer 1. The concession game with buyer 1 may continue until the time $T_1^e = -\log(z_b/A_2)/\lambda_1$. The players' concession game strategies are $F_s^1(t) = 1 - e^{-\lambda_s^1 t}$, $F_1(t) = 1 - A_2 e^{-\lambda_1 t}$, $F_s^2(t) = 1 - e^{-\lambda_s^2 t}$, and $F_2(t) = 1 - z_b(z_b/A_2)^{\frac{\lambda_s^1 \lambda_2}{\lambda_s^2 \lambda_1}} (z_s)^{-\lambda_2/\lambda_s^2} e^{-\lambda_2 t}$.

Remark 4. If $(z_b/A_2)^{\lambda_s^1/\lambda_1} > z_s > (z_b/A_2)^{\lambda_s^1/\lambda_1} z_b^{\lambda_s^2/\lambda_2}$, then the seller is strong against buyer 2 and his equilibrium payoff of visiting buyer 2 first is

$$V_s^2 = \alpha_s \left[1 - z_b \left(\frac{(z_b/A_2)^{\frac{\lambda_s^1}{\lambda_1}}}{z_s} \right)^{\frac{\lambda_2}{\lambda_s^2}} \right] + \alpha_2 z_b \left(\frac{(z_b/A_2)^{\frac{\lambda_s^1}{\lambda_1}}}{z_s} \right)^{\frac{\lambda_2}{\lambda_s^2}} \quad (9)$$

Proposition 3. Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 2 first and the players' initial bids satisfy $\alpha_2 > \delta\alpha_1$ but fail to satisfy (??). The continuation strategies will be as follows. The rational seller never leaves buyer 2 and the concession game ends by the time $T_2^e = \min \left\{ -\frac{\log z_b}{\lambda_2}, -\frac{\log z_s}{\lambda_s^2} \right\}$ for sure, if the game has not yet ended. The players' concession game strategies are $F_s^2(t) = 1 - z_s e^{\lambda_s^2(T_2^e - t)}$ and $F_2(t) = 1 - z_b e^{\lambda_2(T_2^e - t)}$. If the seller leaves 2 at time T_2^e and goes to 1, then buyer 1 will immediately concede to the seller believing that he is the commitment type.

4.2 The Gap Between the Buyers' Bids is Sufficiently Big (i.e., $\delta\alpha_1 \geq \alpha_2$)

In this subsection, I characterize the equilibrium strategies where the players bids in the first stage satisfy $1 > \alpha_s > \alpha_1 > \alpha_2 > 0$ and $\delta\alpha_1 \geq \alpha_2$. I furthermore assume that in equilibrium, if the seller is indifferent between conceding to his current negotiating partner and abandoning it to visit the other buyer, the rational seller will choose to abandon his partner. This assumption is binding only when $\alpha_2 = \delta\alpha_1$. There are infinitely many equilibria when this equality holds. This restriction implies that among all possible equilibria in this particular case, we select the one that yields the highest payoff to the seller. It gives the highest payoff because the seller does not go back and forth between the buyers and lose his surplus to search cost with no additional benefit. Therefore, this particular assumption in this particular case does not affect the results of this paper.

Proposition 4. Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 2 first and the players' initial bids satisfy $\delta\alpha_1 \geq \alpha_2$. The continuation strategies will be as follows. The rational seller makes a "take it or leave it" offer to buyer 2 and, if not accepted, goes directly to buyer 1. Rational buyer 2 immediately accepts the seller's demand and finishes the game at time 0 with probability 1. In case buyer 2 does not concede to the seller, the seller infers that buyer 2 is the commitment type, and so he never comes back to this buyer again. The concession game with buyer 1 may continue until the time $T_1^e = \min \left\{ -\frac{\log z_s}{\lambda_s^1}, -\frac{\log z_b}{\lambda_1} \right\}$ with the following strategies: $F_1(t) = 1 - z_b e^{\lambda_1(T_1^e - t)}$ and $F_s^1(t) = 1 - z_s e^{\lambda_s^1(T_1^e - t)}$ for all $t \geq 0$.

Remark 5. If $\delta\alpha_1 \geq \alpha_2$, then the seller is strong against buyer 2 and his equilibrium payoff of visiting buyer 2 first is

$$V_s^2 = \alpha_s(1 - z_b) + \delta z_b [(1 - z_b e^{\lambda_1 T_1^e})\alpha_s + z_b e^{\lambda_1 T_1^e} \alpha_1] \quad (10)$$

Proposition 5. Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 1 first and the players' initial bids satisfy $\delta\alpha_1 \geq \alpha_2$ and $z_b \geq \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)}$. The continuation strategies will be as follows. The rational seller never leaves 1 and the concession game ends by the time $T_1^e = \min \left\{ -\frac{\log z_b}{\lambda_1}, -\frac{\log z_s}{\lambda_s^1} \right\}$ for sure, if the game has not yet ended. The players' concession game strategies are $F_s^1(t) = 1 - z_s e^{\lambda_s^1(T_1^e - t)}$ and $F_1(t) = 1 - z_b e^{\lambda_1(T_1^e - t)}$. If the seller leaves 1 at time T_1^e and goes to 2, then buyer 2 will immediately concede to the seller believing that he is the commitment type.

Remark 6. If $\delta\alpha_1 \geq \alpha_2$ and $z_b \geq \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)}$ hold, then the rational seller's equilibrium payoff of visiting buyer 1 first is

$$V_s^1 = (1 - z_b e^{\lambda_1 T_1^e})\alpha_s + z_b e^{\lambda_1 T_1^e} \alpha_1 \quad (11)$$

Thus, the seller is strong against buyer 1 if and only if $T_1^e = -\frac{\log z_s}{\lambda_s^1}$ (equivalently $z_s > z_b^{\lambda_1/\lambda_s^1}$).

Proposition 6. Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 1 first and the players' initial bids satisfy $\delta\alpha_1 \geq \alpha_2$, $\frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)} > z_b$ and $z_s^{\lambda_1/\lambda_s^1} \geq z_b$. The continuation strategies will be as follows. Rational buyer 1 immediately accepts the seller's demand with certainty upon his arrival. Otherwise, the seller leaves 1 immediately at time 0 (knowing that buyer 1 is obstinate), and goes directly to buyer 2. Rational buyer 2 instantly accepts the seller's demand with probability 1 upon the seller's arrival. In case buyer 2 does not concede, the rational seller immediately leaves this buyer, directly returns to buyer 1, accepts 1's demand α_1 and finalizes the game.

Remark 7. If $\delta\alpha_1 \geq \alpha_2$, $\frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)} > z_b$ and $z_s^{\lambda_1/\lambda_s^1} \geq z_b$ hold, then the seller is strong against buyer 1 and his equilibrium payoff of visiting buyer 1 first is

$$V_s^1 = (1 - z_b)\alpha_s[1 + \delta z_b] + \delta^2 z_b^2 \alpha_1 \quad (12)$$

Proposition 7. Consider a sequential equilibrium strategy, following a history where the rational seller visits buyer 1 first and the players' initial bids satisfy $\delta\alpha_1 \geq \alpha_2$ and $\frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)} > z_b > z_s^{\lambda_1/\lambda_s^1}$. The continuation strategies will be as follows. Rational buyer 1 does not concede to the seller's demand at time 0. The seller leaves 1 immediately at time 0 (still believing that buyer 1 is the obstinate type with probability z_b), and goes directly to 2. Rational buyer 2 instantly accepts the seller's demand with probability 1 upon the seller's arrival. In case buyer 2 does not concede, the rational seller immediately leaves this buyer, directly returns to buyer 1. With the return of the seller, buyer 1 and the seller immediately starts playing the concession game, which

may continue until the time $T_1^e = -\frac{\log z_b}{\lambda_1}$ with the following strategies: $F_1(t) = 1 - e^{-\lambda_1 t}$ and $F_s^1(t) = 1 - (z_s/z_b^{\lambda_s/\lambda_1})e^{-\lambda_s t}$ for all $t \geq 0$.

Remark 8. If $\delta\alpha_1 \geq \alpha_2$ and $\frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)} > z_b > z_s^{\lambda_1/\alpha_s^1}$ hold, then the seller is weak against buyer 1. However, the rational seller's equilibrium payoff of visiting buyer 1 first is not α_1 . In particular, it is

$$V_s^1 = \delta \left[(1 - z_b)\alpha_s + \delta z_b \left[(1 - z_b e^{\lambda_1 T_1^e})\alpha_s + z_b e^{\lambda_1 T_1^e} \alpha_1 \right] \right] \quad (13)$$

5. CLOSING REMARKS

This paper studies the equilibrium of a three-stage, continuous-time competitive multilateral bargaining game between a single seller and two buyers. The buyers compete in a Bertrand fashion and make initial posted-price offers simultaneously. The seller can accept one of these offers costlessly, or else visit one of the buyers and try to bargain for a higher price. Each of the three players suspects that his opponents might be a commitment type with some positive probability. The first main result shows that playing the tough bargainer will not help the long side of the market (i.e., the buyers). That is, the unique equilibrium outcome is the Walrasian outcome, and thus it is efficient, where the monopolist seller gets the entire rent and leaves no surplus to the buyers. This is true regardless of the search friction and the players' uncertainties. The key assumption for this result is that the probability of obstinacy is constant and independent of the chosen price/posture. The second main result shows that a buyer's reputational advantage will not soften the competition. In this case, the seller can strengthen his bargaining position against both buyers by picking the "unattractive" buyer, who is tougher or greedier than the other buyer, first and leaving the attractive buyer aside because he starts the sequel with a strong outside option.

APPENDIX

1. Proofs for Section 4

Proof of Propositions 1 and 2. Their proofs are similar to the proof of Proposition 3.1 in the Appendix of Ozyurt (2015). Therefore, they are presented in the Supplementary Appendix.

Proof of Lemma 1. Given the history described in the premises of the Lemma, in equilibrium, the rational seller (weakly) prefers to go to buyer 1 over conceding to buyer 2. In equilibrium, rational buyer 2 anticipates that the seller will never concede to him, and hence accepts α_s at time 0 without any delay. Therefore, if buyer 2 is rational, then the game should finish at time 0. Otherwise, the seller leaves the second buyer at time 0 and directly goes to buyer 1. Therefore, the concession game with buyer 1 ends at time $T_1^e = \tau_s^1 = \min\{\tau_s^1, \tau_1\}$ for sure. The term $\tau_s^1 = \inf\{t \geq 0 \mid \hat{F}_s^1(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda_s^1}$ indicates the time that the seller's reputation would reach 1 if he was playing a strategy \hat{F}_s^1 against

buyer 1 with $\hat{F}_s^1(0) = 0$. Likewise, $\tau_1 = \inf\{t \geq 0 \mid \hat{F}_1(t) = 1 - z_b\} = -\frac{\log z_b}{\lambda_1}$ denotes the time that buyer 1's reputation would reach 1 if he was playing a strategy \hat{F}_1 with $\hat{F}_1(0) = 0$. Given the equilibrium strategies in Propositions 1 and 2, the rest follows.

Proof of Lemma 2. Given the history described in the premises of the Lemma, in equilibrium, the rational seller prefers to play the concession game with buyer 2 over going to buyer 1 at time 0. Note that the rational seller leaves buyer 2 if and only if buyer 2 is the commitment type. The reason for this is the following: Since the players' concession game strategies are increasing and continuous, the buyers' reputation will eventually converge to 1 at some finite time. Similarly, the seller's reputation will increase to a level that is sufficiently high (but strictly less than 1) so that it will be optimal for the seller to visit the other buyer. Hence, in equilibrium, the seller will leave buyer 2 when the seller is indifferent between conceding to buyer 2 and visiting buyer 1. Call this time as T_2^d . Moreover, buyer 2's reputation must reach 1 at time T_2^d . The rational seller will break his indifference at this time by leaving the buyer because according to Proposition 1 concession game strategies must be continuous in their domain, eliminating the possibility of mass acceptance at time T_2^d . Hence, buyer 2's reputation reaches 1 at time $T_2^d = \tau_2 = \min\{\tau_s^2, \tau_2\}$. The term $\tau_s^2 = \inf\{t \geq 0 \mid \hat{F}_s^2(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda_s^2}$ indicates the time that the seller's reputation would reach 1 if he was playing a strategy \hat{F}_s^2 against buyer 2 with $\hat{F}_s^2(0) = 0$. Likewise, $\tau_2 = \inf\{t \geq 0 \mid \hat{F}_2(t) = 1 - z_b\} = -\frac{\log z_b}{\lambda_2}$ denotes the time that buyer 2's reputation would reach 1 if he was playing a strategy \hat{F}_2 with $\hat{F}_2(0) = 0$.

However, leaving 2 is optimal for the rational seller if and only if the seller's reputation at time T_2^d reaches z_s^* , implying that

$$c_s^2 e^{-\lambda_s^2 T_2^d} = \frac{z_s}{z_s^*} \quad (14)$$

Given the value of T_2^d , solving the last equality yields the seller's equilibrium strategy with buyer 2. Finally, the game ends with buyer 1 at time $T_1^e = \tau_s^1 = \min\{\tau_s^1, \tau_1\}$ for sure where $\tau_s^1 = -\frac{\log z_s^*}{\lambda_s^1}$ and $\tau_1 = -\frac{\log z_b}{\lambda_1}$, at which points both players' reputation simultaneously reach 1. Given the value of T_1^e , Propositions 1 and 2 imply the concession game strategies with buyer 1.

Proof of Lemma 3. In equilibrium, the rational seller leaves buyer 2 when his reputation reaches z_s^* , implying that Equation (??) holds. If $c_s^2 = 1$, then $T_2^d = -\frac{\log z_s}{\lambda_s^2} + \frac{\lambda_s^1 \log(z_b/A_2)}{\lambda_1 \lambda_s^2}$, and it is smaller than $-\frac{\log z_b}{\lambda_2}$ as $(z_b/A_2)^{\lambda_s^1/\lambda_1} z_b^{\lambda_s^2/\lambda_2} < z_s$. Similar to Lemma 2, the game ends with buyer 1 at time $T_1^e = \frac{\log z_s^*}{\lambda_s^1}$. Given the values of T_2^d and T_1^e , Propositions 1 and 2 imply the concession game strategies.

Proof of Proposition 3. If condition (??) does not hold, then the rational seller will not be able to build enough reputation so that visiting the other buyer is an optimal action for him. Also, rational seller will not benefit by deviating. The most profitable deviation for the seller is to immediately leaving buyer 2 and visiting buyer 1. With this deviation, rational seller's expected payoff will be at most $\delta[(1 - z_b)\alpha_s + z_b\alpha_1]$ which is less than conceding to buyer 2 (i.e., α_2) as condition (??) does not hold. Therefore, similar to Abreu and Gul (2000), the seller will play the concession game with buyer 2 until both players' reputation simultaneously reach 1. Propositions 1 and 2 give the functional forms of the players' strategies.

Proof of Proposition 4. Since we have $\alpha_2 \leq \delta\alpha_1$, the rational seller prefers going to buyer 1 over conceding to 2 at any given time. That is, in equilibrium, the rational seller never concedes to buyer 2.

Since rational buyer 2 anticipates that the rational seller will never accept his demand in equilibrium, he concedes to the seller with probability 1 upon his arrival without any delay. Thus, the seller leaves buyer 2 immediately if rational buyer 2 does not accept the seller's demand and finish the game at time 0.

When the seller arrives at buyer 1 (after visiting 2), the rational seller and buyer 1 play the concession game until some finite time T_1^e as the seller has no outside option worth leaving buyer 1. As characterized in the proof of Proposition 1, the equilibrium strategies are $F_s^1(t) = 1 - c_s^1 e^{-\lambda_s^1 t}$ and $F_1(t) = 1 - c_1 e^{-\lambda_1 t}$. Therefore, the concession game with buyer 1 ends at time $T_1^e = \min\{\tau_s^1, \tau_1\} = \min\{-\frac{\log z_s}{\lambda_s^1}, -\frac{\log z_b}{\lambda_1}\}$ for sure if it does not end before.

Proof of Proposition 5. If the rational seller concedes to buyer 1, his instantaneous payoff is α_1 . However, if the rational seller leaves buyer 1 at time 0 and goes to buyer 2, then we know from Proposition 4 that rational buyer 2 will immediately accept the seller's demand. Therefore, the rational seller's continuation payoff of leaving buyer 1 at time 0 is $\bar{V}_s = \delta[(1 - z_b)\alpha_s + \delta z_b v_s^1]$, where $v_s^1 = (1 - F_1(0))\alpha_1 + F_1(0)\alpha_s$ denotes the seller's expected payoff in his second visit to buyer 1. In equilibrium v_s^1 must be equal to α_1 . Suppose for a contradiction that $v_s^1 > \alpha_1$. It requires that buyer 1 offers positive probabilistic gift to the seller on his second visit. In this case, buyer 1's expected payoff must be $1 - \alpha_s$ (as $F_s^1(0)F_1(0) = 0$ by Proposition 1). However, optimality of the equilibrium implies that rational buyer 1 should have accepted the seller's offer with probability 1 when the seller attempts to leave him for the first time. Hence, it must be that in equilibrium $v_s^1 = \alpha_1$. As a result, the rational seller's expected payoff if he leaves buyer 1 at time 0 is $\bar{V}_s = \delta[(1 - z_b)\alpha_s + \delta z_b \alpha_1]$.

Finally, if \bar{V}_s is strictly greater than α_1 , then the rational seller prefers leaving buyer 1 immediately at time 0 over conceding to buyer 1. $\bar{V}_s > \alpha_1$ implies that $z_b < \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)}$. Since the opposite of this inequality is assumed to hold, the rational seller will never leave buyer 1. Equilibrium strategies follow from Proposition 1 and 3. Moreover, rational seller will not benefit by deviating and not conceding to buyer 1, leaving 1 at time T_1^e and visiting buyer 2. This is because with this deviation, rational seller's expected payoff will be at most $(1 - z_b)\delta\alpha_s + z_b\delta^2\alpha_1$, and this payoff is less than the payoff of conceding to buyer 1 (i.e., α_1) due to the condition on z_b .

Proof of Proposition 6. Because $z_b < \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)}$, the rational seller prefers to visit buyer 2 instead of conceding to buyer 1. Note that rational seller will revisit buyer 1 if he learns that buyer 2 is the commitment type. On the other hand, rational buyer 1's payoff of conceding to the seller is $1 - \alpha_s$, but buyer 1's payoff of letting the seller leave him and waiting for his second visit is $\delta_b^2 z_b \hat{v}_1$, where $\delta_b = e^{-r_b \Delta}$ and \hat{v}_1 denotes buyer 1's expected payoff of playing the concession game with the seller when he visits buyer 1 for the second time. Notice that $\hat{v}_1 = F_s^1(0)(1 - \alpha_1) + (1 - F_s^1(0))(1 - \alpha_s)$ with $F_s^1(0) = 1 - z_s e^{\lambda_s^1 T_1^e}$, where $T_1^e = \frac{-\log z_b^*}{\lambda_1}$ (which is strictly less than $\frac{-\log z_s}{\lambda_s^1}$), $z_b^* = \frac{z_b}{z_b + (1 - z_b)\mu}$, and μ denotes the probability that rational buyer 1 accepts the seller's demand at time 0 (i.e., the probability of buyer's concession at the seller's first visit to buyer 1).

The equality $1 - \alpha_s \geq \delta_b^2 z_b \hat{v}_1$ implies that we must have $z_b^* \leq \left[(\alpha_s - \alpha_1) z_s / (1 - \alpha_1 - \frac{1 - \alpha_s}{\delta_b^2 z_b}) \right]^{\lambda_1 / \lambda_s^1}$, which is clearly less than $z_s^{\lambda_1 / \lambda_s^1}$. Therefore, buyer 1 will make initial concession with a positive probability when the seller visits his store for the second time, implying that there will be no equilibrium

where buyer 1 is indifferent between conceding to seller at time 0 and letting the seller leave him. Thus, rational buyer 1 will concede to the seller at time 0 with probability 1 whenever $z_b \leq z_s^{\lambda_1/\lambda_s^1}$ holds. Then, the rest of the claim immediately follows.

Proof of Proposition 7. Similar arguments to the proof of Proposition 6 suffices to show that rational buyer 1 will not concede to the seller at time 0 whenever $z_b > z_s^{\lambda_1/\lambda_s^1}$ holds. Because $z_b < \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)}$ holds, the rational seller prefers to visit buyer 2 instead of conceding to buyer 1. Propositions 1 and 3 will give the equilibrium strategies of the concession game played between buyer 1 and the seller conditional on the seller visits buyer 1 for the second time.

2. Proofs for Section 3

Proof of Theorem 1. Suppose for a contradiction that for some z_b, z_s, δ, r_b and r_s there exists a sequential equilibrium strategy profile, where the players' strategies in stage 1 satisfy $1 > \alpha_s > \alpha_1, \alpha_2$. Next, I will show that the following four results (Propositions 1.1-1.4) must simultaneously hold. However, since they are incompatible, we will achieve the desired contradiction. The case where $1 = \alpha_s > \alpha_1, \alpha_2$ will be examined separately at the end of this proof.

Proposition 1.1. In any sequential equilibrium where $1 > \alpha_s > \alpha_1, \alpha_2$ holds, at least one buyer must be weak.

Proof of Proposition 1.1. Suppose for a contradiction that there exists an equilibrium where $1 > \alpha_s > \alpha_1, \alpha_2$ holds and both buyers are strong. Since both buyers are strong, the seller must be weak against both buyers. Thus, the rational seller's expected payoff of visiting buyer i first is simply α_i . That is, the seller's continuation payoff, following a history where the seller visits buyer i first at time 0 to play the concession game, is α_i .

If the last argument is true, then the buyers' prices must be the same (i.e., $\alpha_1 = \alpha_2 = \alpha_b < \alpha_s$). To prove this claim I need to consider two cases:

CASE 1: Suppose for a contradiction that $\alpha_1 > \alpha_2$ and $\delta\alpha_1 \geq \alpha_2$. The seller is weak against buyer 1 if and only if premises of Proposition 5 and $z_b > z_s^{\lambda_1/\lambda_s^1}$ hold. According to Proposition 5, rational seller never leaves buyer 1 in equilibrium, which implies that buyer 2's expected payoff in the game is strictly less than $1 - \alpha_s$ (indeed it is $z_b z_s (1 - \alpha_s) e^{r_b(T_1^d + \Delta)}$.) However, rational buyer 2 can profitably deviate by posting $\alpha_s + \epsilon$ in stage 1 for some small $\epsilon \geq 0$, contradicting the optimality of the equilibrium.

CASE 2: Suppose for a contradiction that $\alpha_1 > \alpha_2$ and $\delta\alpha_1 < \alpha_2$. Since $\alpha_1 > \alpha_2$, the rational seller will visit buyer 1 first (and get the expected payoff of α_1). According to Proposition 2, the rational seller visits buyer 2 when he builds enough reputation for obstinacy so that he becomes strong against buyer 2. Hence, Buyer 2's expected payoff in the game must be strictly less than $1 - \alpha_s$. Similar arguments in the previous case shows that buyer 2 can profitably deviate, contradicting the optimality of the equilibrium.

Since both cases lead to a contradiction, we can conclude that both buyers must post the same price, say $\alpha_b < \alpha_s$. Since the seller is weak against both buyers and both buyers bid the same price in stage 1, then one of the followings must hold: (1) $z_s < z_b^{\lambda_s/\lambda_b}$ (as given by Proposition 3) holds if $\delta\alpha_s \leq \alpha_b$ or if $z_b \geq A$ holds, or (2) $z_s < (z_b^2/A)^{\lambda_s/\lambda_b}$ (as given by Lemma 2) holds if the inequalities $\delta\alpha_s > \alpha_b$ and $z_b < A$ are satisfied. Note that, we have $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha_b}$, $\lambda_b = \frac{\alpha_b r_s}{\alpha_s-\alpha_b}$ and $A = \frac{\delta\alpha_s-\alpha_b}{\delta(\alpha_s-\alpha_b)}$. Because both buyers are strong, the above inequalities of z_b and z_s are strict.

CASE 1: If the first inequality (i.e., $z_s < z_b^{\lambda_s/\lambda_b}$) holds, then I will show that the buyers can profitably deviate, which contradicts with the optimality of the equilibrium. First note that the seller must be indifferent between the buyers as both post the same price. Let σ_i be the probability that the seller visits buyer i first according to the equilibrium strategy. If $\sigma_i < \sigma_j$, then I say buyer i has more incentive to deviate. The buyers' incentive to deviate is equal if $\sigma_i = 1/2$. Therefore, pick the buyer i such that $\sigma_i \leq \sigma_j$. Suppose, w.l.o.g, that it is buyer 1. Instead of posting α_b , he would post $\alpha_1 = \alpha_b + \epsilon$ where $\epsilon > 0$ is small enough so that $\alpha_1 > \delta\alpha_s$ and $\alpha_b > \delta\alpha_1$. With these parameter values, according to Proposition 3, rational seller never leaves the buyer he visits first. Moreover, since ϵ can be selected very small, we can guarantee that $z_s < z_b^{\lambda_s/\lambda_b} < z_b^{\lambda_i/\lambda_i}$ as $\lambda_s/\lambda_b > \lambda_s^1/\lambda_1$, where $\lambda_s^i = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha_i}$, $\lambda_i = \frac{\alpha_i r_s}{\alpha_s-\alpha_i}$. That is, the seller will still be weak against both buyers. As a result, the seller will pick buyer 1 who posts a higher price in stage 1 with probability 1.

Next, I need to show that with this deviation buyer 1 will get a higher expected payoff. The rational buyer's expected payoff before deviation is $V_1 = \sigma_1 v_1 + (1 - \sigma_1) z_s z_b e^{r_b(T_2^d + \Delta)} (1 - \alpha_s)$ where $T_2^d = \frac{-\log z_b}{\lambda_b}$ and $v_1 = (1 - \alpha_b)(1 - z_s z_b^{-\lambda_s/\lambda_b}) + (1 - \alpha_s) z_s z_b^{-\lambda_s/\lambda_b}$, which can be calculated by Proposition 3. The second part of V_1 corresponds to the buyer 1's expected payoff when the seller visits buyer 2 first. Note that according to Proposition 3, the rational buyer 2 will never let the seller leave his store and the rational seller will never leave buyer 2 either. On the other hand, buyer 1's expected payoff after deviating to $\alpha_1 = \alpha_b + \epsilon$ is $\hat{V}_1 = (1 - \alpha_b - \epsilon)(1 - z_s z_b^{-\lambda_s^1/\lambda_1}) + (1 - \alpha_s) z_s z_b^{-\lambda_s^1/\lambda_1}$. Since $\sigma_1 < 1$ and v_1 is continuous at α_b , for any $\hat{\epsilon} > 0$ satisfying $\hat{V}_1 - z_s z_b e^{r_b(T_2^d + \Delta)} (1 - \alpha_s) = \hat{\epsilon}$ we can find a small enough $\epsilon > 0$ so that $|\hat{V}_1 - v_1| < \hat{\epsilon}(1 - \sigma_1)/\sigma_1$ so that we have $\hat{V}_1 - V_1 > 0$ as desired.

CASE 2: If the second inequality (i.e., $z_s < (z_b^2/A)^{\lambda_s/\lambda_b}$) holds, then it must be true that we have $\delta\alpha_s > \alpha_b$ and $z_b < A$. Next, I will show that the buyers can profitably deviate from α_b , which contradicts with the optimality of the equilibrium. Suppose, w.l.o.g, that the buyer who has higher incentive to deviate is buyer 1 (i.e., $\sigma_1 \leq \sigma_2$). Instead of posting α_b , he would post $\alpha_1 = \alpha_b + \epsilon$ where $\epsilon > 0$ small enough so that the followings hold; (1) $\delta\alpha_s > \alpha_1 > \alpha_b$, and (2) $z_b < A_i = \frac{\delta\alpha_s - \alpha_i}{\delta(\alpha_s - \alpha_j)}$; and (3) $z_s < (z_b/A_i)^{\lambda_s^j/\lambda_j} z_b^{\lambda_s^i/\lambda_i}$ for each $i = 1, 2$ and $j \in \{1, 2\}$ with $j \neq i$. We can always pick $\epsilon > 0$ small enough so that these three inequalities simultaneously hold. As a result of this deviation, the seller will still be weak against both buyers, and hence, optimality of the equilibrium implies that the seller will pick the buyer who posts the higher price (i.e., buyer 1) for sure. Similar arguments used in Case 1 suffice to show that rational buyer 1 can gain by deviating to $\alpha_b + \epsilon$. Because we attain contradiction in both cases 1 and 2, we can conclude that at least one buyer must be weak in equilibrium.

Proposition 1.2. In any sequential equilibrium where $1 > \alpha_s > \alpha_1, \alpha_2$ holds, if a buyer is weak, then the buyers' prices, α_1 and α_2 , are different (i.e., $\alpha_1 \neq \alpha_2$).

Proof of Proposition 1.2. Suppose for a contradiction that there exists an equilibrium where $1 > \alpha_s > \alpha_1, \alpha_2$ holds, one of the buyers is weak and the buyers post the same price α_b . Now, I will show that the seller must visit the weak buyer, say buyer 1, with probability 1. I suppose for a contradiction that the seller visits buyer 1 with probability $\sigma_1 < 1$. Therefore, rational buyer 1's expected payoff in the game is less than $\sigma_1(1 - \alpha_s) + (1 - \sigma_1)z_b\delta(1 - \alpha_s)$, where the first part is buyer 1's expected payoff when he is visited first and the second part is strictly greater than his expected payoff if he is visited as second (recall that rational buyer 2 never lets the seller leave him without an agreement.) That is, rational buyer 1's expected payoff in the game is strictly less than $1 - \alpha_s$. However, by posting his price as α_s (if $\alpha_b < \alpha_s$) or $\alpha_s + \epsilon$ where $\epsilon > 0$ is small enough if $\alpha_b = \alpha_s$, buyer 1 can ensure strictly higher payoff. Note that both buyers must be weak in equilibrium because the buyers are identical and they both post the same price. Thus, in equilibrium, the seller must visit both buyers with probability 1, leading to contradiction.

Proposition 1.3. In any sequential equilibrium where $1 > \alpha_s > \alpha_1 > \alpha_2$ holds, buyer 1 must be strong.

Proof of Proposition 1.3. Suppose for a contradiction that there exists an equilibrium where $1 > \alpha_s > \alpha_1 > \alpha_2$ holds and buyer 1 is weak. Next, I will show that buyer 2 has an incentive to deviate, contradicting the optimality of equilibrium. There are two exhaustive cases we need to consider:

CASE I ($\delta\alpha_1 \leq \alpha_2$): Similar arguments used in Proposition 1.2 suffices to show that the seller must visit buyer 1 with probability 1 (or else buyer 1 can deviate and post a price α_s to guarantee higher expected payoff in the game). Therefore, in equilibrium the second buyer will be visited after the seller visits 1. Moreover, since $\delta\alpha_1 \leq \alpha_2$, the rational seller leaves buyer 1 only if he is strong against buyer 2 in the concession game they play after the seller visits buyer 1. Hence, buyer 2's expected payoff in the game is strictly less than $z_b(1 - \alpha_s)$. However, rational buyer 2 can ensure the payoff of $1 - \alpha_s$ by posting the price of α_s .

CASE II ($\delta\alpha_1 > \alpha_2$): There are two subcases we should consider.

Case II–A: Suppose that $z_b \leq z_s^{\lambda_1/\lambda_s^1}$ or $z_b \geq \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)}$. In this case, according to Propositions 5 and 6, buyer 1 will concede to the seller at the time he visits him at time 0. Therefore, similar arguments used in Proposition 1.2 suffices to show that the seller must visit buyer 1 with probability 1. Similar arguments used in Case I ensures that the second buyer has an incentive to deviate.

Case II–B: Suppose that $z_s^{\lambda_1/\lambda_s^1} < z_b < \frac{\delta\alpha_s - \alpha_1}{\delta(\alpha_s - \delta\alpha_1)}$. Then, buyer 1 is weak but buyer 1 does not concede to the seller at the time the seller visits him first at time 0. Therefore, the seller does not have to visit buyer 1 first with probability 1. However, since $z_b > 0$, we must have $\delta\alpha_s > \alpha_1$ (i.e., α_1 and α_s are apart from one another). Therefore, by Proposition 7, the seller's expected payoff of visiting buyer 1 is $V_s^1 = \delta[(1 - z_b)\alpha_s + \delta z_b \alpha_1]$, and by Proposition 4 the seller's expected payoff of visiting buyer 2 first is $V_s^2 = (1 - z_b)\alpha_s + \delta z_b \alpha_1$. Clearly, $V_s^2 > V_s^1$. That is, the seller must visit buyer 2 first in equilibrium.

Since the seller visits buyer 2 first with probability 1, Proposition 7 implies that rational buyer 2's expected payoff in the game is $1 - \alpha_s$. However, by deviating to a price $\hat{\alpha}_2 = \alpha_s - \epsilon$ where $\epsilon > 0$ is small enough, buyer 2 can increase his payoff. Here is why: Since $z_b > z_s^{\lambda_1/\lambda_s^1} > z_s^{\hat{\lambda}_2/\hat{\lambda}_s^2}$ as $\frac{\lambda_1}{\lambda_s^1} = \frac{\alpha_1 r_s}{(1 - \alpha_s) r_b} <$

$\frac{\hat{\alpha}_2 r_s}{(1-\alpha_s)r_b} = \frac{\hat{\lambda}_2}{\hat{\lambda}_s^2}$ and ϵ is sufficiently small so that $\delta\alpha_s < \hat{\alpha}_2$ (and therefore $z_b > 0 > \frac{\delta\alpha_s - \hat{\alpha}_2}{\delta(\alpha_s - \delta\hat{\alpha}_2)}$), then by Proposition 5, rational seller never leaves buyer 2, and thus, the seller is weak against buyer 2. Since buyer 2 becomes strong, he will achieve an expected payoff that is a combination of $1 - \alpha_s$ and $1 - \hat{\alpha}_2$ (as buyer 2's expected payoff is $F_s^2(0)(1 - \hat{\alpha}_2) + (1 - F_s^2(0))(1 - \alpha_s)$ and $F_s^2(0) > 0$ as buyer 2 is strong). However, this deviation is profitable for buyer 2 when the seller visits buyer 2 first (once he deviates to $\hat{\alpha}_2$) with a sufficiently high probability. Buyer 2 can ensure that the seller visits his store with probability 1 if he picks $\epsilon > 0$ small enough so that $V_s^2 = \hat{\alpha}_2 > V_s^1$, where $V_s^1 = \alpha_s(1 - z_b) + \delta z_b \hat{\alpha}_2$ (by Proposition 4). That is, if $\epsilon < \alpha_s \frac{z_b(1-\delta)}{1-\delta z_b}$ holds, then buyer 2 can profitably deviate from α_b .

Proposition 1.4. In any sequential equilibrium where $1 > \alpha_s > \alpha_1 > \alpha_2$ holds, buyer 2 is strong.

Proof of Proposition 1.4. Suppose for a contradiction that there exists an equilibrium where $1 > \alpha_s > \alpha_1 > \alpha_2$ holds and buyer 2 is weak. Similar arguments in the proof of Proposition 1.2 ensure that the seller must visit buyer 2 with probability 1 in equilibrium, and so, buyer 2's expected payoff in the game is $1 - \alpha_s$. Then, there are two exhaustive cases we need to consider.

CASE I ($\delta\alpha_1 < \alpha_2$): According to Lemmas 1-3 or Proposition 3, in equilibrium, buyer 1 must concede with a positive probability at the beginning of the concession game between the seller and buyer 1. Furthermore, because the seller visits buyer 1 only if buyer 2 is a commitment type, the equations in (??) imply that buyer 1's equilibrium payoff is less than $z_b(1 - \alpha_s)$.¹⁹ However, buyer 1 can deviate to a price $\hat{\alpha}_1 = \alpha_s + \epsilon$ where $\epsilon \geq 0$ and achieve a payoff that is very close to $1 - \alpha_s$.

CASE II ($\delta\alpha_1 \geq \alpha_2$): There are three exhaustive sub-cases that we need to consider:

Case II-A ($z_s^{\lambda_1/\lambda_s^1} \geq z_b$): According to Proposition 5 or Proposition 6, buyer 1's expected payoff in the game will be less than $z_b(1 - \alpha_s)$ because the seller visits 1 after visiting buyer 2 first. However, buyer 1 can profitably deviate by posting a price $\hat{\alpha}_1 = \alpha_s + \epsilon$ where $\epsilon \geq 0$ is small enough, which contradicts with the optimality of the equilibrium.

Case II-B ($\alpha_1 \geq \delta\alpha_s$ and $z_s^{\lambda_1/\lambda_s^1} < z_b$): I will show that buyer 2 can achieve a higher expected payoff by deviating to a price $\hat{\alpha}_2 = \alpha_1 + \epsilon$, where $\epsilon > 0$ is small enough so that both $\hat{\alpha}_2 > \delta\alpha_s$ and $\alpha_1 > \delta\hat{\alpha}_2$ hold (that is α_1 , $\hat{\alpha}_2$ and α_s are close to each other). Following this deviation, by Proposition 3, the rational seller never leaves the buyers, and both buyers are strong in the game because $z_s^{\hat{\lambda}_2/\hat{\lambda}_s^2} < z_s^{\lambda_1/\lambda_s^1} < z_b$. Thus, the seller will prefer to visit buyer 2 first because $\hat{\alpha}_2 > \alpha_1$. Moreover, since buyer 2 is strong in the game once he deviates to $\hat{\alpha}_2$, his expected payoff in the game will be $[1 - F_s^2(0)](1 - \alpha_s) + F_s^2(0)(1 - \hat{\alpha}_2)$ that is strictly higher than $1 - \alpha_s$ as $F_s^2(0) > 0$. Therefore, buyer 2 can profitably deviate from α_2 , contradicting the optimality of equilibrium.

Case II-C ($\alpha_1 < \delta\alpha_s$ and $z_s^{\lambda_1/\lambda_s^1} < z_b$): I will show that buyer 2 can achieve a higher expected payoff by deviating to a price $\hat{\alpha}_2 = \alpha_s - \epsilon$, where $\epsilon > 0$ is sufficiently small so that both $\alpha_1 < \delta\hat{\alpha}_2$ and $\delta\alpha_s < \hat{\alpha}_2$ hold. As a result of this deviation, Proposition 5 ensures that the rational seller never leaves buyer 2 if he visits him first, and the seller is weak against buyer 2. Moreover, if buyer 2 picks ϵ small enough so that $V_s^2 = \hat{\alpha}_2$ is larger than $V_s^1 = \alpha_s(1 - z_b) + \delta z_b \hat{\alpha}_2$ (by Proposition 4) (i.e., $\epsilon < \alpha_s \frac{z_b(1-\delta)}{1-\delta z_b}$),

¹⁹Note that buyer 1's expected payoff must be discounted with the time the seller spends with buyer 2.

then the rational seller prefers to visit buyer 2 first. As a result, buyer 2 guarantees an expected payoff slightly higher than $1 - \alpha_s$ by deviating from α_2 , contradicting the optimality of equilibrium.

We reach the desired contradictions in all of these exhaustive cases, finalizing the proof of Proposition 1.4.

Finally, I will briefly discuss that there does not exist an equilibrium where $1 = \alpha_s > \alpha_1, \alpha_2$. Suppose for a contradiction that such an equilibrium exists. The first observation will be that the rational buyers' equilibrium payoffs must be positive. If, for example, rational buyer 1's equilibrium payoff is 0, then he would profitably deviate. If buyer 1 deviates to a price $\hat{\alpha}_1 = 1 - \epsilon$ where ϵ is small enough so that $\hat{\alpha}_1 > \delta\alpha_s$, then by Proposition 3, the seller never leaves buyer 1 and buyer 1 becomes strong. Moreover, if ϵ is small enough, then the rational seller prefers to visit buyer 1 first. This is true because the rational seller's payoff of visiting buyer 1 (i.e., $V_s^1 = 1 - \epsilon$) is higher than his payoff of visiting buyer 2, V_s^2 that is at most $(1 - z_b) + z_b v$ where $v \equiv \max\{\delta, \alpha_2\} \in (0, 1)$, for sufficiently small values of ϵ (in particular $\epsilon < z_b(1 - v)$).

However, if the rational seller accepts a buyer's demand (at some time) with a positive probability, then the buyer's best response is not to concede to the seller. This is true simply because if a buyer concedes to the seller whose posted price is 1, then his payoff will be 0. Therefore, in the equilibrium where $1 = \alpha_s > \alpha_1, \alpha_2$, the seller must be conceding to both buyers with positive probabilities and the buyers never concede to the seller. Therefore, the seller must be weak against both buyers. Optimality of equilibrium implies that the seller should concede to the buyer with the highest price, or the buyers' prices are the same (i.e., $\alpha_1 = \alpha_2$). In either case, similar arguments in the proof of Proposition 1.1 would show that, at least one of the buyers has an incentive to overbid his opponent unless $\alpha_i = 1$ for some $i \in \{1, 2\}$, contradicting the optimality of equilibrium.

Proof of Theorem 2. Note that we have $z_1 > z_2$, $A = \frac{\delta\alpha_s - \alpha_b}{\delta(\alpha_s - \alpha_b)}$, and $\lambda_s/\lambda_b = \frac{(1 - \alpha_s)r_b}{\alpha_b r_s}$. Consider the following four exhaustive cases regarding the value of z_s .

Case I: $\left(\frac{z_1 z_2}{A}\right)^{\lambda_s/\lambda_b} \geq z_s$. In this case, by Lemma 2 (or Remark 3) we know that the seller's expected payoff of visiting each buyer is α_b . Therefore, the seller is indifferent between choosing buyer 1 and buyer 2.

Case II: $\left(\frac{z_2}{A}\right)^{\lambda_s/\lambda_b} > z_s > \left(\frac{z_1 z_2}{A}\right)^{\lambda_s/\lambda_b}$. By Lemma 3 (or Remark 4), the seller's expected payoff of visiting buyer 1 and 2 first are

$$V_s^1 = V_s^2 = \alpha_s \left[1 - \frac{z_1 z_2}{A z_s^{\lambda_b/\lambda_s}} \right] + \alpha_b \left[\frac{z_1 z_2}{A z_s^{\lambda_b/\lambda_s}} \right]. \quad (15)$$

Therefore, the seller is indifferent between visiting buyer 1 and buyer 2 first.

Case III: $\left(\frac{z_1}{A}\right)^{\lambda_s/\lambda_b} > z_s \geq \left(\frac{z_2}{A}\right)^{\lambda_s/\lambda_b}$. By lemma 1 (or Remark 2), the seller's expected payoff of visiting buyer 1 first is

$$V_s^1 = \left\{ (1 - z_1) + \delta z_1 \left(1 - \frac{z_2}{z_s^{\lambda_b/\lambda_s}} \right) \right\} \alpha_s + \frac{\delta z_1 z_2}{z_s^{\lambda_b/\lambda_s}} \alpha_b. \quad (16)$$

and visiting buyer 2 first is given in Equation (??). One can check that we have $V_s^2 \geq V_s^1$ if and only if $z_s \geq \left(\frac{z_2}{A}\right)^{\lambda_s/\lambda_b}$. Hence, the seller prefers to visit buyer 2 first.

Case IV: $z_s \geq \left(\frac{z_1}{A}\right)^{\lambda_s/\lambda_b}$. By lemma 1 (or Remark 2) again the seller's expected payoff of visiting buyer 1 is given by Equation (??) while buyer 2 first is

$$V_s^2 = \left\{ (1 - z_2) + \delta z_2 \left(1 - \frac{z_1}{z_s^{\lambda_b/\lambda_s}}\right) \right\} \alpha_s + \frac{\delta z_1 z_2}{z_s^{\lambda_b/\lambda_s}} \alpha_b.$$

Because $\delta < 1$, we have $V_s^1 < V_s^2$. Therefore, the seller prefers to visit buyer 2 first.

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