Take-It-or-Leave-It Offers in Negotiations: Behavioral Types and Endogenous Deadlines*

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Abstract

This paper studies a reputational bargaining model, and investigates the effects of "*exit threat*" on negotiators' equilibrium behaviors. Although it is a purely cheap talk message, exit threat is effective and has two main effects: (1) it renders the final outcome efficient and unique, and (2) shifts the bargaining power towards the negotiator who can make this threat. Setting a deadline for negotiations pressures the opponent and incentivizes her to compromise. However, a deadline that is too early makes the opponent less willing to compromise. Thus, effective deadline is uniquely determined. Last minute agreements occur with a positive probability if negotiators cannot reach an immediate agreement. Frequency of agreement has peaks at the beginning and at the end of negotiations (deadline effect), and is flat otherwise.

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1. Introduction

In episode 14 (season 6) of the famous TV series *House*, the main story revolves around the last eight hours of a critical negotiation between Lisa Cuddy (the dean of Medicine) and a contract negotiator from Atlantic Net Insurance to renew a contract. Dr. Cuddy and the contract negotiator have been arguing about the contract for eight months, and that day, Dr. Cuddy lays it all on the line. When they meet at 8:30 a.m., Dr. Cuddy makes her final offer that

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she agrees to capitated structure but wants a 12% increase in rates. The contract negotiator refuses Dr. Cuddy's offer immediately. Then she tells him that this is the hospital's final offer, and he has until 3:00 p.m. to agree, or she will make a public announcement that they are no longer accepting Atlantic Net. At noon, Dr. Cuddy tracks down the CEO of Atlantic Net at lunch and confronts him about the contract. He blows her off and tells her that her tactic will not work. The negotiator from Atlantic Net returns around 2pm and offers an 8% increase as their final offer, but Dr. Cuddy declines and wants the full 12%. After a few stressful hours of waiting, the story ends with a good news. The negotiator revisits Dr. Cuddy just before her announcement and tells that the company has agreed to her 12% proposal.

Negotiators often use *take-it-or-leave-it* offers as a final strategic maneuver in order to push their rivals towards more acceptable terms.[1] It usually begins with a milder threat such as "this is my final offer," and if it does not work, escalates to a higher level: "take this one or I am calling off the negotiation" (Ma, Yang and Savani 2019). Namely, subtext of a take-it-or-leave-it offer contains a threat of not making further concessions (i.e., *commitment threat*), and depending on the negotiator's position, a threat of leaving the bargaining table (i.e., *exit threat*). For example, both firms and unions may make a take-it-or-leave-it offer in collective bargaining, but unions' threats usually follow an announcement for a strike.[2] This paper investigates impacts of these bluffs on negotiators' behaviors.

Threats are effective if they are credible, so I follow a reputational approach. I study a stylized four-stage, infinite-horizon, continuous-time bargaining game that I adapt from the reputational bargaining literature (see, for example, Özyurt 2015a & 2015b, Abreu and Gul 2000, and Kambe 1999). The novel twist is that a negotiator can bluff about ending the negotiation, by announcing a deadline. There are four defining features of the model. First, two negotiators bargain over the division of a unit surplus and begin with announcing their demands (final offers). Second, Negotiator 1 declares a deadline for the game if the demands are incompatible. Third, each negotiator faces some small uncertainty that her opponent may be a behavioral type, who acts on her threats. Fourth, the negotiation phase adopts a war of attrition protocol, during which each agent chooses between conceding (yielding the opponent), waiting for the opponent's concession, and leaving the game. Potentially small but positive uncertainty regarding opponent's true type provides strong incentive for negotiators to build reputation on their commitments, affecting their equilibrium behavior.

This exercise is important for three reasons. First, ability to exit the game and announcing it in advance creates endogenous deadline effect, and it is absent in the reputational bargaining literature.[3] Second, exit threats are usually studied in isolation from commitment threats, and

[1]See, for example, Sugden, Wang and Zizzo (2019).

[2]In labor union negotiations, the term *Boulwarism*, named after former vice president of General Electrics Lemuel Boulware, refers to a negotiating tactic that does not permit any further revisions. See, for example, Falcao (2016).

[3]Compte and Jehiel (2002) allow negotiators to "*exit*" (i.e., *end the game without a concession*) at any time they want; but exit is an action that is more favorable than concession. In Özyurt (2015 a&b) a negotiator (buyer) can go back and forth between two opponents (sellers). By doing so she builds her reputation on obstinacy, and the value of her outside option. Nevertheless, the buyer does not have an option to exit the

the current work is one of the very few that combines the two. Third, equilibrium behaviors (strategies) are essentially unique, and provide interesting testable hypothesis and explanations for some widely noted experimental observations in the literature.

Unlike the standard multiplicity and inefficiency results in the literature (e.g., Kambe 1999), equilibrium outcome is unique and efficient due to the endogenous deadline effect. Exit threat has a potential to make Negotiator 1 advantageous, but Negotiator 2 can always render her threat ineffective by committing to a demand very close to Negotiator 1's residual share. Thus, unique optimal demands are such that Negotiator 1's share is the maximum she can get while her exit threat cannot be neutralized by her opponent. In equilibrium, negotiators play a mixed strategy where they are indifferent between conceding and waiting at all times before the deadline. Other things being equal, announcing a shorter deadline may disincentivize the opponent for concession, and incentivize for letting negotiation end with a disagreement. This is true because aggressive deadlines may signal that Negotiator 1 is not likely to act on her threat. Settlement rate has peaks at time zero and at deadline, but it is flat otherwise.

2. The Framework

Two negotiators, 1 and 2, bargain over the division of a unit surplus. Negotiators discount time, and $r_i > 0$ denotes rate of time preference for Negotiator $i \in \{1, 2\}$. **Reputational bargaining game**, denoted by G, is modeled by the following four-stage, infinite-horizon, and continuous-time game:

All four stages begin, and the first three stages end at time 0. No discounting applies between these four stages. In Stage 1 each negotiator independently and simultaneously announces a share of the unit surplus $x_i \in (0, 1)$, denoting Negotiator i's final offer (or demand). Interpretation is that Stage 1 is the time in which negotiators make their commitment threats. If these demands are compatible (i.e., $x_1 + x_2 \leq 1$), then the game is over in Stage 1, and one of the two divisions of surplus $(x_1, 1 - x_1)$ or $(1 - x_2, x_2)$ is implemented with a probability of 1/2 each. If the demands are incompatible (i.e., $x_1 + x_2 > 1$), then the game moves to the next stage. In Stage 2 Negotiator 1 sends a cheap talk message by announcing a deadline: Time that she intends to leave the bargaining table and end the game.[4] Negotiator 2 does not take any action in this stage. Stage 2 is interpreted as the time in which Negotiator 1 makes her exit threat. In Stage 3 each negotiator learns whether she is allowed to continue the game; Negotiator i is replaced with a "*behavioral type*" with a probability $z_i \in (0, 1)$, and allowed to stay in the game with probability $1 - z_i$. In Stage 4 (still time 0) the following continuous-time concession game begins: At any given time $t \geq 0$, a negotiator either leaves the bargaining table (i.e., exits the game),

game. Negotiators in Ekmekci and Zhang (2022) can exit the game, if they mutually agree, and enjoy external resolution opportunities, but exit times cannot be communicated in advance. Fanning (2016) studies deadline effect in a standard reputational bargaining framework, but deadline is exogenous for negotiators.

[4]Stage 2 announcement is a cheap talk message in the sense that the game does not end at the announced deadline, and Stage 4 continues forever (i.e., beyond the announced deadline) if both negotiators choose to do so.

or accepts her opponent's Stage 1 demand, or waits for her concession. If a negotiator concedes or leaves the negotiation table, then the game ends.

Because the concession game is in continuous time, there may occur some measure-theoretic pathologies associated with negotiators exiting the game and conceding at a given time. I resolve such potential issues in the manner introduced by Abreu and Pearce (2007). In particular, for any time $t \geq 0$ of Stage 4, corresponding to the "conventional time" t, I suppose two logically consecutive stages, t^1 and t^2, of time t. No discounting applies between these "two stages." A negotiator can concede to her opponent at both stages, t^1 and t^2, but can exit the game only at the first stage, t^1, of time t.

Negotiators are risk neutral. If the game ends at time $t \geq 0$ with Negotiator i's concession, then the payoffs to Negotiator i and j are $(1 - x_j)e^{-r_i t}$ and $x_j e^{-r_j t}$, respectively. If i concedes and j exits in Stage t^1 of time t, then the game ends with i's concession. In case of simultaneous concession, one of the two divisions of surplus, $(x_1, 1 - x_1)$ or $(1 - x_2, x_2)$, is implemented with a probability of 1/2 each. If both negotiators wait forever or a negotiator exits the game, then both receive a payoff of zero.[5] A negotiator who is replaced with a behavioral type also receives a payoff of zero. Negotiators maximize the expected discounted values of their shares. The entire structure of the reputational bargaining game G is common knowledge.

Strategies of the Behavioral Types: A behavioral type simply commits to the announcements of her predecessor. That is, behavioral type of Negotiator 1 never concedes, and she exits the game at the announced deadline. Behavioral type of Negotiator 2 never concedes and never exits the game.

Strategies of the (Rational) Negotiators: A negotiator's strategy must describe her choice of demand and her decision about when (if ever) to concede to her opponent and exit the game, given demands and deadline. Negotiator 1's strategy must also describe her choice of a deadline, given demands.

More formally, Stage 1 strategy of Negotiator i is a pure action $x_i \in (0, 1)$. Let $x \equiv (x_1, x_2)$ denote the negotiators' Stage 1 demands and $\mathbb{D} \equiv \{(x_1, x_2) \in (0, 1)^2 | \; x_1 + x_2 > 1\}$ be the set of incompatible demands. Stage 2 strategy of Negotiator 1 is a function $K : \mathbb{D} \to \overline{\mathbb{R}}_+$, where $K(x) = \infty$ is interpreted as not announcing a deadline.[6] Given a history $(x, k) \in \mathbb{D} \times \overline{\mathbb{R}}_+$, Stage 4 strategy for Negotiator i is a right-continuous distribution function $F_i^{k,x} : \overline{\mathbb{R}}_+ \to [0, 1]$, representing the probability of Negotiator i conceding to Negotiator j by conventional time t (inclusive of both Stage t^1 and t^2 of time t). Therefore, $F_i^{k,x}(t^1)$ denotes the probability of Negotiator i conceding to j by time t inclusive of Stage t^1 of time t, and $F_i^{k,x}(t) = F_i^{k,x}(t^2)$.

Negotiator i makes zero payoff when she leaves the bargaining table. However, she can make a positive payoff in any equilibrium by conceding to her opponent since $x_j \in (0, 1)$. Thus, *rational negotiators never exit the game in equilibrium.* For this reason and for simplicity, I ignore negotiators' exit strategies.

[5] Exiting the game is an inefficient outcome, for much the same reasons as strikes in collective bargaining. It represents committing oneself to an irreversible course, also known as *burning bridges*.

[6] We have $\overline{\mathbb{R}}_+ \equiv [0, \infty) \cup \{\infty\}$.

Payoffs: Given the strategies $F_1^{k,x}$ and $F_2^{k,x}$, let $B_i^{k,x} : \overline{\mathbb{R}}_+ \to [0, 1 - z_i]$ denote Negotiator j's belief about Negotiator i accepting x_j and finishing the game prior to time t. Thus, $B_i^{k,x}(t) = (1 - z_i)F_i^{k,x}(t)$ since behavioral types never concede.

Given a history (x, k), Negotiator i's **interim payoff** —payoff after learning that she will continue the game— of conceding to Negotiator j at time $t \leq k$, conditional that the game has not yet over, is

$$U_i(t, F_j^{k,x}) = (1 - x_j)\left[1 - B_j^{k,x}(t)\right] e^{-r_i t} + x_i \int_0^t e^{-r_i y} dB_j^{k,x}(y) + \frac{1}{2}(1 + x_i - x_j)\left[B_j^{k,x}(t) - B_j^{k,x}(t^-)\right] e^{-r_i t}, \tag{1}$$

where $B_j^{k,x}(t^-) = (1 - z_j)\lim_{y \nearrow t} F_j^{k,x}(y)$. Negotiator i's **ex-ante payoff** —payoff before learning Stage 3 outcome— is

$$U_i^e(F_i^{k,x}, F_j^{k,x}) = (1 - z_i)\int_0^\infty U_i(y, F_j^{k,x}) dF_i^{k,x}(y). \tag{2}$$

Equilibrium refers to Perfect Bayesian Nash equilibrium.

Like Crawford (1982), Kambe (1999), Wolitzky (2012), and Ellingsen and Miettinen (2014), probability of commitment (i.e., z_i's) is independent of the announcements. There are two motivations for this. First, many real-world negotiations agree with this approach: Negotiators may (have to) commit to their demands or threats, regardless of how unpleasant it may be, depending on how events unfold during negotiation. For example, a state leader may commit to her announcements if revoking her commitments turns out to be a very costly action (such as cost of losing face or credibility of her rhetoric), and the leader may not know the size of these costs for a fact before seeing the reactions of her constituency. The second motivation has a technical rationale. Abreu and Gul (2000) interpret behavioral types as irrational players that are born with their commitments. Given this interpretation, if negotiator i is rational and demanding x_i, then this is her strategic choice. If she is a behavioral type, then she merely declares the demand corresponding to her type. One can easily extend this approach to the current setup. However, it would substantially complicate the model when there are multiple types.

3. Formal Results

This section characterizes equilibrium of the game G by working backward. If the negotiators' demands are compatible, then the game ends in Stage 1. Interesting cases occur when demands are incompatible. Therefore, I fix the negotiators' demands $x = (x_1, x_2)$ for the rest of this section, and assume that $x \in \mathbb{D}$. Given that Negotiator 1's deadline announcement is k, what are the equilibrium strategies $F_1^{k,x}$ and $F_2^{k,x}$ of Stage 4? This question is answered next.

The following functions play crucial role describing the equilibrium strategies:

$$\lambda_1(x) \equiv \frac{r_2(1-x_1)}{x_1+x_2-1}, \quad \lambda_2(x) \equiv \frac{r_1(1-x_2)}{x_1+x_2-1}, \quad T(x) \equiv \min\left\{-\frac{\ln z_1}{\lambda_1(x)}, -\frac{\ln z_2}{\lambda_2(x)}\right\},$$

and

$$Z(x) \equiv \frac{x_1+x_2-1}{x_2}, \quad K^*(x) \equiv \frac{\ln\left(\frac{Z(x)}{z_1}\right)}{\lambda_1(x)}.$$

Theorem 1. *In equilibrium, after a history $(x,k) \in \mathbb{D} \times \overline{\mathbb{R}}_+$, the game G ends before or at time k. Namely, $F_i^{k,x}(t) = F_i^{k,x}(k)$ for all $t \geq k$ and all $i \in \{1,2\}$. Furthermore,*

1. *if $k = 0$, then*
   - *(i) $F_1^{k,x}(0^1) = 0$, $F_1^{k,x}(0^2) = 1$, and $F_2^{k,x}(0^1) = 1$ whenever $Z(x) < z_1$,*
   - *(ii) $F_2^{k,x}(0^1) = F_2^{k,x}(0^2) = 0$, $0 \leq F_1^{k,x}(0^1) \leq 2\left(\frac{Z(x)-z_1}{Z(x)(1-z_1)}\right)$, and $F_1^{k,x}(0^2) = 1$ whenever $z_1 < Z(x)$, and*
   - *(iii) $F_1^{k,x}(0^1) = 0$, $F_1^{k,x}(0^2) = 1$, $0 \leq F_2^{k,x}(0^1) \leq 1$, and $F_2^{k,x}(0^2) = F_2^{k,x}(0^1)$ whenever $z_1 = Z(x)$.*

2. *if $0 < k < \min\{K^*(x), T(x)\}$, then*
   - *(i) $F_1^{k,x}(t) = \frac{1}{1-z_1}\left(1 - \frac{z_1}{Z(x)}e^{\lambda_1(x)(k^1-t)}\right)$ whenever $t \in [0,k^1]$, and $F_1^{k,x}(k) = 1$, and*
   - *(ii) $F_2^{k,x}(t) = \frac{1}{1-z_2}\left(1 - e^{-\lambda_2(x)t}\right)$ whenever $t \in [0,k^1]$, and $F_2^{k,x}(k) = F_2^{k,x}(k^1)$.*

3. *if $k = K^*(x) < T(x)$, then*
   - *(i) $F_1^{k,x}(t) = \frac{1}{1-z_1}\left(1 - e^{-\lambda_1(x)t}\right)$ whenever $t \in [0,k^1]$, and $F_1^{k,x}(k) = 1$,*
   - *(ii) $F_2^{k,x}(t) = \frac{1}{1-z_2}\left(1 - c_2^{k,x}e^{-\lambda_2(x)t}\right)$ where $c_2^{k,x} \in \left[z_2\left(\frac{Z(x)}{z_1}\right)^{\lambda_2(x)/\lambda_1(x)}, 1\right]$ whenever $t \in [0,k^1]$, and $F_2^{k,x}(k) = F_2^{k,x}(k^1)$.*

4. *if $K^*(x) < k < T(x)$, then*
   - *(i) $F_1^{k,x}(t) = \frac{1}{1-z_1}\left(1 - e^{-\lambda_1(x)t}\right)$ whenever $t \in [0,k^1]$, and $F_1^{k,x}(k) = 1$, and*
   - *(ii) $F_2^{k,x}(t) = \frac{1}{1-z_2}\left(1 - z_2e^{\lambda_2(x)(k^1-t)}\right)$ whenever $t \in [0,k^1]$, and $F_2^{k,x}(k) = F_2^{k,x}(k^1)$.*

5. *if $T(x) \leq k$, then for each $i \in \{1,2\}$, $F_i^{k,x}(t) = \frac{1}{1-z_i}\left(1 - z_ie^{\lambda_i(x)(T(x)-t)}\right)$ whenever $t \in [0,T(x)]$, and $F_i^{k,x}(t) = 1$ whenever $t > T(x)$.*

Although equilibrium behaviors characterized in Theorem 1 seem convoluted, the basic intuition is straightforward. Fix a history $(x,k) \in \mathbb{D} \times \overline{\mathbb{R}}_+$ where $k > 0$. Negotiator i's behavior in Stage 4 is a probability distribution over concession times,

$$F_i^{k,x}(t) = Pr(\text{Negotiator } i \text{ concedes prior to time } t).$$

Let $\lambda_i(x,k,t)$ be her instantaneous concession (or hazard) rate at time t. By definition, it must satisfy

$$\lambda_i(x,k,t) = \frac{dB_i^{k,x}(t)/dt}{1 - B_i^{k,x}(t)}. \tag{3}$$

Concession game has no equilibrium in pure strategies, and so the negotiators must mix between conceding and waiting at all times. Negotiator j would be indifferent between conceding at time t and waiting for an infinitesimal period Δ and then conceding at time $t+\Delta$ if and only if

$$(1-x_i)e^{-r_j t} = x_j e^{-r_j t}\lambda_i(x,k,t)\Delta + [1-\lambda_i(x,k,t)\Delta](1-x_i)e^{-r_j(t+\Delta)},$$

where $\lambda_i(x,k,t)\Delta$ indicates the probability that Negotiator i concedes during Δ. Solving the last equation for $\lambda_i(x,k,t)$ and taking its limit as Δ approaches 0 yields the hazard rate, which is independent of time t and deadline k:

$$\lambda_i(x) = \frac{r_j(1-x_i)}{x_1 + x_2 - 1}. \tag{4}$$

Using (3) and integrating up the hazard rate (4) yield

$$F_i^{k,x}(t) = \frac{1}{1-z_i}\left(1 - c_i^{k,x}e^{-\lambda_i(x)t}\right),$$

where $c_i^{k,x} = 1 - F_i^{k,x}(0)$. Therefore, Stage 4 strategies grow with fixed hazard rates, $\lambda_1(x)$ and $\lambda_2(x)$. The announced deadline, k, determines the initial concession probabilities, $F_1^{k,x}(0)$ and $F_2^{k,x}(0)$, which give rise to five cases of Theorem 1.

To understand those five cases, first consider the conventional time $t = k \geq 0$. In Stage k^2 of time k, optimal strategy for Negotiator 1 is to accept x_2 and end the game because her rationality is public information in this stage. Now consider Stage k^1 of time k. Assuming that Negotiator 1 is not expected to make concession in this stage, conceding yields the payoff of $(1-x_1)$ to Negotiator 2. However, not conceding yields expected payoff of $(1-\hat{z}_1(k^1))x_2$, where $\hat{z}_1(k^1)$ is Negotiator 2's posterior belief of Negotiator 1 being the behavioral type.[7] Therefore, Negotiator 2 is indifferent between conceding and waiting in Stage k^1 of time k if and only if $\hat{z}_1(k^1) = \frac{x_1+x_2-1}{x_2}$, which is equivalent to $Z(x)$. The last observation is sufficient to pin down the equilibrium strategies when $k=0$. If Negotiator 1's initial reputation, z_1, is higher than $Z(x)$, then Negotiator 2 must accept x_1 and finish the game in Stage 0^1 of time 0 (case 1.(i) of Theorem 1). If $z_1 < Z(x)$, then Negotiator 2 never concedes.

Two other thresholds, $K^*(x)$ and $T(x)$, become critical in determining the size of initial concessions when k is positive. Given that Negotiator i is not expected to concede at time $t=0$ and that the announced deadline is not binding (e.g., $k=\infty$), the term $-\frac{\ln z_i}{\lambda_i(x)}$ indicates the time at which Negotiator i's reputation would reach one, if the game ever reaches this point. Once Negotiator i's reputation reaches one, her rational opponent has no incentive to wait. Thus,

[7] Negotiator i's reputation at time t, given her Stage 4 strategy $F_i^{k,x}$, is calculated by Bayes' rule, so $\hat{z}_i(t) = \frac{z_i}{1-B_i^{k,x}(t)}$.

$T(x)$ denotes the time at which both negotiators' reputations would reach one in equilibrium. Announcing a deadline beyond this point (i.e., $k \geq T(x)$) is equivalent to not making an exit threat because it has no impact on equilibrium strategies (case 5 of Theorem 1).

Finally, $K^*(x)$ is the time required for Negotiator 1 to build her reputation to the level of $Z(x)$ if she is not expected to make concession at time zero. If her announced deadline k is later than this time (i.e., $k > K^*(x)$), then her initial concession will be zero and Negotiator 2 will make positive initial concession, size of which depends on k (cases 3 and 4 of Theorem 1). However, if Negotiator 1 announces $k \leq K^*(x)$, then she must make initial concession with a positive probability, and the choice of k determines the size of her own concession (case 2 of Theorem 1).

Now we are ready to characterize the optimal deadline.

Theorem 2. *In equilibrium, the optimal deadline $K(x)$ is effectively unique and satisfies*

$$K(x) = \begin{cases} \infty, & \text{if } z_1 < \underline{Z}(x) \\ K^*(x), & \text{if } \underline{Z}(x) \leq z_1 < Z(x) \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where $\underline{Z}(x) \equiv Z(x) z_2^{\lambda_1(x)/\lambda_2(x)}$.

The term $\underline{Z}(x)$ is the level for Negotiator 1's initial reputation, z_1, that makes $T(x)$ and $K^*(x)$ equal. Theorem 2 implies that if Negotiator 1's initial reputation is sufficiently low (in particular $z_1 < \underline{Z}(x)$), then exit threat is ineffective: Optimal deadline for such values of z_1 is announcing anything higher than $T(x)$, all of which yield the same payoffs to the negotiators as those that they would get if Negotiator 1 does not set a deadline. Thus, $K(x) = \infty$ is effectively the unique optimal strategy. For all other cases, *exit threat is effective*. When Negotiator 1's initial reputation is sufficiently high (i.e., $z_1 \geq \underline{Z}(x)$), then optimal deadline is 0, so Negotiator 1 turns the concession game into an ultimatum game. For the intermediate values of z_1, the optimal deadline is uniquely determined by the function $K^*(x)$.

Since we now know the optimal strategies of Stage 2 and 4, we are ready to characterize equilibrium demands of Stage 1. Define

$$\alpha^* \equiv \sup \left\{ \alpha_1 \in (0,1) \middle| \ \underline{Z}(\alpha_1, \alpha_2) \leq z_1 \text{ for all } \alpha_2 \in [1-\alpha_1, 1) \right\}$$

to be the highest demand Negotiator 1 can select without jeopardizing the effectiveness of her exit threat. Namely, if Negotiator 1 selects a demand higher than α^*, then Negotiator 2 can always find a profitable deviation; a demand that would make Negotiator 1's exit threat ineffective.

Theorem 3. *In any equilibrium of the game G, Negotiator 1 demands α^*, Negotiator 2 demands $1 - \alpha^*$, and the game ends in Stage 1.*

4. Overview of the Results and Comparison with the Empirical Findings

This section summarizes important aspects of the equilibrium behaviors, and compares them with empirical and experimental findings in the literature. Experiments cited below are not designed to test the current model; they consider exogenous deadlines, yet it is endogenous in my model. With this caveat in mind, Cramton and Tracy (1992) [CT henceforth] is probably the most relevant study. They consider union contract negotiations, where unions correspond to Negotiator 1 and strikes —threats that are inefficient for all parties and that only union can make— correspond to the exit threat in my model.[8] CT show that daily settlement rate has a peak right after the expiration date (i.e., time 0) and then decreases gradually with time. After the first week, the weekly settlement rate from dispute is roughly constant at 11 %. These observations agree with the equilibrium behaviors, and I discuss this in detail below.

Assuming that the negotiators' demands are incompatible (i.e., $x \in \mathbb{D}$) and the announced deadline k is positive and not ineffective (i.e., $0 < k < T(x)$), Stage 4 equilibrium behaviors satisfy the following:

1. **Delay in agreements:** Negotiators prolong the agreement all the way to the announced deadline with a positive probability, though delay is costly, because they can build reputation on their commitments. Higher reputation that negotiators get by waiting compensates the cost of delay, and gives enough incentive to postpone the agreement.[9]

2. **Optimal Deadline:** Effectiveness of the exit threat fades away (i.e., the optimal deadline becomes ∞) as (1) Negotiator 1's initial reputation z_1 decreases, or (2) her demand x_1 increases, or (3) she becomes more impatient, or (4) Negotiator 2 gets more patient, or (5) z_2 increases.

3. **Likelihood of disagreements:** Rational Negotiator 2 may let the game end with a disagreement if Negotiator 1 **undershoots** the target deadline (i.e., her announced deadline is smaller than the optimal level, $k < K(x)$). The **likelihood of disagreement** is inversely related to k; it approaches one as k approaches zero. This finding is supported by Karagözoğlu and Kocher (2019) but contradicts with some of the earlier research on time pressure effects on bargaining (see, for example, Roth et al. 1988; and De Dreu et al. 2000). These earlier papers claim that high time pressure (by an earlier deadline) induces lower resistance to conceding, and so individuals under high time pressure might care less about their own position and are more willing to compromise. This does not hold in my model because undershooting the target deadline is an "immature bluffing," signaling that Negotiator 1 is not likely to act on her threat. Thus, Negotiator 2 is more likely to call Negotiator 1's bluff and less willing to compromise as k decreases to zero.

[8]CT call expiration date of the old contracts by deadline. These dates correspond to the beginning of Stage 4 (time 0) in my model because expiration date is the earliest time that a union can start a strike. A deadline in my model, however, corresponds to the beginning of a strike.

[9]Delay in reputational (Abreu-Gul) bargaining setting is experimentally supported by Embrey et al. (2015) and Malik et al. (2021).

4. **Initial shock and deadline effects:** Concession game strategies are continuous and have no atoms except the beginning and the deadline. Each negotiator concedes with a constant hazard rate. Concessions at time 0 and at the deadline are called **initial concession** and **final concession**, respectively.

   (a) **Initial concession effect:** Existence of a strong initial concession effect, which is empirically supported by Cramton and Tracy (1992) and Güth et al. (2005), is always consistent with equilibrium. Negotiator 1 makes initial concession with a positive probability if she undershoots the target deadline. Negotiator 2 makes initial concession with a positive probability if the deadline is more than the target (i.e., $k > K(x)$, in which case I say Negotiator 1 **overshoots** the target deadline). Probability of initial concession increases as deadline k approaches zero.

   (b) **Deadline effect (last-moment agreements):** Negotiator 1 always makes a final concession with a positive probability, whenever the game reaches the deadline. This observation agrees with a large body of empirical evidence on deadline effects (see, for example, Güth et al. 2005 and Roth et al. 1988). Negotiator 2 never makes a final concession, and so observable deadline effects are always due by Negotiator 1.

5. **Intensity of the deadline effect:** Probability of final concession (or the deadline effect) is independent of the announced deadline k if Negotiator 1 undershoots the target. Otherwise, it decreases with the deadline (see Figure 1):

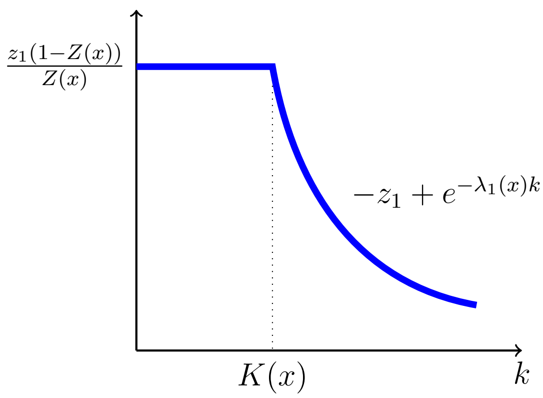

Figure 1: Probability of deadline effect as a function of the announced deadline

This finding agrees with Karagözoğlu and Kocher (2019) where they report that frequency of last-moment agreements is higher in high time pressure treatment than in low time pressure treatment.

If Negotiator 1 undershoots the deadline, then she knows that her bluff was immature, so her opponent will call her bluff with a positive probability by letting the game end with a disagreement. Therefore, backing down earlier with a higher probability is better for Negotiator 1 than waiting and backing down at the very last moment. As a result of this, frequency of last-moment agreements does not change with the announced deadline. However, if Negotiator 1 overshoots the deadline, then her exit threat is credible in the sense that Negotiator 2 will never let the negotiation end with a disagreement. Therefore, Negotiator 1 prefers to delay her concession. But as announced deadline increases, frequency of last-moment agreements decreases because she has more time (or opportunity) to concede before the deadline.

6. **Settlement rate:** For an outsider, who observes the announced deadline and demands, but not the negotiators' types, settlement rate (i.e., probability that Stage 4 ends by a concession of a negotiator) *at time* $t \in (0, k)$ is the sum of the instantaneous concession rates, $\lambda_1(x) + \lambda_2(x)$. The settlement rate has two peaks, one at time zero and the other at the deadline, and is flat otherwise. The latter peak is not as high as the former (see Figure 2).

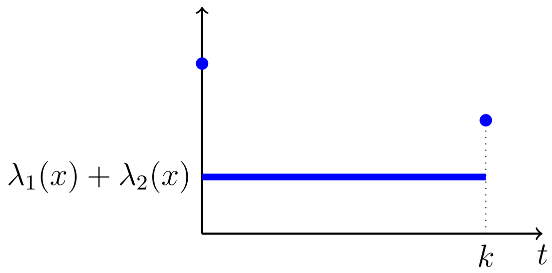

Figure 2: Settlement rate at time t.

These observations agree with the findings of Cramton and Tracy (1992), given the transformation discussed at the beginning, and Güth et al. (2005), where they show that settlement rate has one peak at the beginning and another at the end, and mostly flat in between.

Equilibrium outcome is unique and efficient. The equilibrium share α^* does not have a closed-form solution, but numerical simulations indicate that Negotiator 1 significantly benefits from having the option to announce a deadline, though it is a purely cheap talk message. Assuming that negotiators are ex-ante identical (i.e., $z_i = z$ and $r_i = r$ for $i = 1, 2$), equal share of the unit surplus would be an equilibrium, absent Stage 2 (see, Kambe 1999). However, α^* takes values 0.788, 0.699, and 0.638 (approximately) when the exogenous probability of the behavioral types, z, is 0.4, 0.1, and 0.01, respectively. Although its convergence is slower than z, α^* converges 0.5 in the limit when z approaches zero.

5. Concluding Remarks and Related Literature

Commitment and exit threats are studied, to some extent, separately in the bargaining literature. Schelling (1966) points out the potential benefits of commitment in strategic and dynamic environments and asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. Crawford (1982), Muthoo (1996), and Ellingsen and Miettinen (2008) follow this approach and show that commitment can be rationalized in equilibrium if revoking commitments is costly. The bargaining models in these papers are one-shot simultaneous-move games. Myerson (1991), Kambe (1999), Abreu and Gul (2000) and Fanning (2022) follow a reputational approach: Parallel to Kreps and Wilson (1982) and Milgrom and Roberts (1982), commitments are modeled as behavioral types that exist in society so that rational players can mimic if they like to do so.[10]

The exit threat is also studied in the bargaining literature. Among many others, Osborne and Rubinstein (1990), Vislie (1988), Shaked (1994), and Ponsati and Sakovics (1998 & 2001) model exit as the ability of opting out of negotiation and receiving an outside option. On the other hand, Fershtman and Seidmann (1993), Ma and Manove (1993), and Ponsati (1995), for example, model exit as a predetermined deadline for the negotiations. The treatment of the exit threat in the current paper has resemblance to both approaches in the sense that announcing an exit time creates a deadline effect, and the ability of choosing exit time provides strategic advantage for much the same reason as the ability of opting out does. Two important differences of the current paper, however, are that (1) rational negotiators do not have to exit the game, unless it is what they want to, and could continue negotiation beyond the deadline, and (2) opting out is an inefficient outcome for all, so it is never optimal to exit.

Fudenberg and Tirole (1986), Chatterjee and Samuelson (1987), and Ponsati and Sakovics (1995) study war of attrition (WOA) games with two-sided uncertainty. Kambe (1999) and Abreu and Gul (2000) take a step forward and add a pre-play to standard WOA games, where the negotiators simultaneously choose their demands, which determine their strategies in the WOA phase. The current paper adds additional layer to these papers by allowing one player to announce a deadline, at which point her behavioral type may end the game.

APPENDIX

I begin with proving the following result, which I extensively use to prove Theorem 1.

Proposition 1. *In equilibrium, after any history $(x, k) \in \mathbb{D} \times \overline{\mathbb{R}}_+$ of the game G where $k \in (0, T(x))$, Stage 4 strategies satisfy the following:*

1. $F_i^{k,x}(t) = F_i^{k,x}(k)$ *for all* $t \geq k$ *and* $i \in \{1, 2\}$. *In particular,* $F_1^{k,x}(k^2) = 1$ *and* $F_2^{k,x}(k) = F_2(k^1)$.
2. $F_i^{k,x}(t) = \frac{1}{1-z_i}(1 - c_i^{k,x} e^{-\lambda_i(x)t})$ *whenever* $t \in [0, k^1]$, *where* $c_i^{k,x} \in [z_i, 1]$ *with* $(1 - c_1^{k,x})(1 - c_2^{k,x}) = 0$.
3. *Negotiator 1's reputation reaches* $Z(x)$ *at time* k^1; *namely* $\hat{z}_1(k^1) = Z(x)$ *whenever* $\hat{z}_2(k^1) < 1$.

[10] Abreu and Sethi (2003) supports the existence of behavioral types from an evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such behavioral types is not compatible with evolutionary stability in bargaining environments. Send and Serena (2022) study eBay Best Offer Bargaining Data and document when and how “insistence” arises in bilateral bargainings.

Proof of Proposition 1. Assume that $x \in \mathbb{D}$ and $k \in (0, T(x))$, and suppose $F_1^{k,x}$ and $F_2^{k,x}$ are part of an equilibrium.

Proof of (*1*): Suppose that the game reaches time k. Behavioral type of Negotiator 1 will certainly exit the game at Stage k^1 of time k. If Negotiator 1 stays in the game, then Negotiator 2 will realize that Negotiator 1 is the rational type. Therefore, starting from Stage k^2 of time k, the continuation game turns into a war of attrition game either with (1) one sided incomplete information, where Negotiator 1 is known to be rational and Negotiator 2's type is unknown, or with (2) complete information, where Negotiator 1 is known to be rational and Negotiator 2 is known to be behavioral. In either case, the unique equilibrium of this continuation game is such that rational Negotiator 2 never concedes and rational Negotiator 1 immediately accepts x_2 and ends the game in Stage k^2 of time k. This is a well-established result in the reputational bargaining literature (first noted by Myerson (1999), then by Abreu and Gul 2000, and more explicitly proved by Fanning 2016 (Lemma 1)), so I skip the formal proof. Therefore, the concession game ends by the announced deadline k. In particular, it must be that $F_1^{k,x}(k^2) = 1$ and $F_2^{k,x}(k) = F_2(k^1)$.

Proof of (*1*): Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and are analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

Lemma A.1. *If $F_i^{k,x}$ is constant on some interval $[t, t'] \subseteq [0, k^1)$, then $F_j^{k,x}$, where $i, j \in \{1, 2\}$ and $j \neq i$, is also constant over $[t, t' + \eta]$ for some $\eta > 0$.*

Lemma A.2. *$F_1^{k,x}$ and $F_2^{k,x}$ do not have a mass point over $(0, k^1)$.*

Lemma A.3. *$F_1^{k,x}(0)F_2^{k,x}(0) = 0$.*

There is no interval (t', t'') with $0 \leq t' < t'' < k^1$ such that both $F_1^{k,x}$ and $F_2^{k,x}$ are constant. Assume on the contrary that $t^* < k^1$ is the supremum of the upper bounds of t'''s such that both $F_1^{k,x}$ and $F_2^{k,x}$ are constant. By Lemma A.1, $F_j^{k,x}$ is constant on $(t', t^* + \eta)$ for some $\eta > 0$ because $F_i^{k,x}$ is constant on (t', t^*). Thus, both $F_1^{k,x}$ and $F_2^{k,x}$ are constant on this later interval, contradicting the definition of t^*. Therefore, $F_1^{k,x}$ and $F_2^{k,x}$ must be strictly increasing over $[0, k^1]$. Lemma A.2 implies the strategies are continuous over $[0, k^1)$. To prove that they are continuous on $[0, k^1]$ as well, suppose for a contradiction that $F_2^{k,x}$ has a jump at time k^1. Then Negotiator 1 prefers to wait for some time before k^1 and concede at time k^2. However, it contradicts the fact that $F_1^{k,x}$ is strictly increasing over $[0, k^1]$. Likewise, $F_1^{k,x}$ cannot have a jump at time k^1. To prove the last claim, suppose for a contradiction that $F_1^{k,x}(k^1) - F_1^{k,x}((k^1)^-) = p > 0$. Then, we have

$$U_2(k^1-\Delta, F_1^{k,x}) = (1-x_1)(1-B_1^{k,x}(k^1-\Delta))e^{-r_2(k^1-\Delta)} + x_2\int_0^{k^1-\Delta} e^{-r_2 y}dB_1^{k,x}(y),$$

$$U_2(k^1, F_1^{k,x}) = \left[\frac{1}{2}(1+x_2-x_1)p + (1-x_1)(1-B_1^{k,x}(k^1))\right]e^{-r_2 k^1} + x_2\int_0^{k^1} e^{-r_2 y}dB_1^{k,x}(y).$$

Therefore, $U_2(k^1, F_1^{k,x}) - U_2(k^1-\Delta, F_1^{k,x}) > 0$ for small values of Δ because this difference is equal to $o_1(\Delta) + o_2(\Delta) + \frac{1}{2}p(1+x_2-x_1)e^{-r_2 k^1}$ where $o_1(\Delta) = x_2\int_{k^1-\Delta}^{k^1} e^{-r_2 y}dB_1^{k,x}(y)$, $o_2(\Delta) = (1-x_1)[(1-B_1^{k,x}(k^1)) - (1-B_1^{k,x}(k^1-\Delta))e^{r_2\Delta}]$ and both $o_1(\Delta)$ and $o_2(\Delta)$ approach 0 as Δ approaches 0. In conclusion, if $F_1^{k,x}$ has a jump at time k^1, then Negotiator 2 prefers to wait for some time $[k^1-\Delta, k^1)$, where $\Delta > 0$ is small, and then concede at time k^1, contradicting that $F_2^{k,x}$ is not constant over $[0, k^1]$. Hence, $F_1^{k,x}$ and $F_2^{k,x}$ must be continuous on $[0, k^1]$. Given the functional form of $U_i(t, .)$, $U_i(t, F_j^{k,x})$ must be continuous as well.

Then, it follows that $D^i \equiv \{t | U_i(t, F_j^{k,x}) = \max_{s\in[0,k^1]} U_i(s, F_j^{k,x})\}$ is dense in $[0, k^1]$. Hence, $U_i(t, F_j^{k,x})$ is constant for all $t \in [0, k^1]$. Consequently, $D^i = [0, k^1]$. Therefore, $U_i(t, F_j^{k,x})$ is differentiable as a function of t. The differentiability of $F_1^{k,x}$ and $F_2^{k,x}$ follows from the differentiability of the utility functions on $[0, k^1]$. Differentiating the utility functions and applying the Leibnitz's rule yields the functional forms as required.

Proof of (*3*): Assume that $\hat{z}_2(k^1) < 1$. In equilibrium, Negotiator 2 is indifferent between conceding and waiting at time k^1 because $F_2^{k,x}$ is continuous on $[0, k^1]$. If she concedes at time k^1, then her instantaneous payoff will be $1-x_1$. If she waits instead, then her expected payoff will be $(1-\hat{z}_1(k^1))x_2$ by (*1.*) above. These two payoffs are equal if and only if $\hat{z}_1(k^1) = Z(x)$, as required. ■

Proof of Theorem 1. Fix a history $(x, k) \in \mathbb{D} \times \overline{\mathbb{R}}_+$ of the game G. Assume that $F_i^{k,x}$ is Stage 4 equilibrium strategy for $i = 1, 2$. Proposition 1 already establishes that $F_i^{k,x}(t) = F_i^{k,x}(k)$ for all $t \geq k$ and $i \in \{1, 2\}$.

Proof of (*1*): Assume $k = 0$. It must be that

$$U_1(0^1, F_2^{k,x}) = F_2^{k,x}(0^1)(1-z_2)\left(\frac{1+x_1-x_2}{2}\right) + z_2(1-x_2) + (1-z_2)(1-F_2^{k,x}(0^1))(1-x_2),$$

$$U_1(0^2, F_2^{k,x}) = (1-z_2)F_2^{k,x}(0^1)x_1 + [(1-z_2)(1-F_2^{k,x}(0^1)) + z_2](1-x_2),$$

$$U_2(0^1, F_1^{k,x}) = F_1^{k,x}(0^1)\left(\frac{1+x_2-x_1}{2}\right)(1-z_1) + z_1(1-x_1) + (1-z_1)(1-F_1^{k,x}(0^1))(1-x_1), \text{and}$$

$$U_2(\infty, F_1^{k,x}) = (1-z_1)F_1^{k,x}(0^1)x_2 + (1-z_1)(1-F_1^{k,x}(0^1))x_2.$$

where $U_2(\infty, F_1^{k,x})$ denote Negotiator 2's expected payoff of never conceding.

Proof of (*i*): Suppose $Z(x) < z_1$. $F_2^{k,x}$ is a best response to $F_1^{k,x}$ because $U_2(\infty, F_1^{k,x}) \leq U_2(0^1, F_1^{k,x})$ iff $(1-z_1)x_2 \leq z_1(1-x_1) + (1-z_1)(1-x_1)$ iff $Z(x) \leq z_1$. Similarly, $F_1^{k,x}$ is a best response to $F_2^{k,x}$ because $U_1(0^2, F_2^{k,x}) = x_1(1-z_2) + z_2(1-x_2) > U_1(0^1, F_2^{k,x}) = z_2(1-x_2) + (1-z_2)(\frac{1+x_1-x_2}{2})$ iff $x_1+x_2 > 1$. To establish uniqueness, first observe that $U_1(0^2, F_2^{k,x}) > U_1(0^1, F_2^{k,x})$ (or $F_1^{k,x}(0^1) = 0$) whenever $F_2^{k,x}(0^1) > 0$. However, if $F_1^{k,x}(0^1) = 0$, then Negotiator 2 prefers to concede at Stage 0^1 of time 0. Therefore, we must show it is never the case that $F_2^{k,x}(0^1) = 0$. Suppose, for a contradiction, that $F_2^{k,x}(0^1) = 0$. It must be that $U_2(F_1^{k,x}) \geq U_2(0^1, F_1^{k,x})$ iff $F_1^{k,x}(0^1) \leq 2(\frac{Z(x)-z_1}{Z(x)(1-z_1)})$, which never holds since $z_1 > Z(x)$.

Proof of (ii): Suppose $z_1 < Z(x)$. Given the range of $F_1^{k,x}$, $U_2(\infty, F_1^{k,x}) \geq U_2(0^1, F_1^{k,x})$, and so $F_2^{k,x}$ is a best response. Likewise, given $F_2^{k,x}$, $U_1(0^1, F_2^{k,x}) = U_1(0^2, F_2^{k,x}) = 1 - x_2$, and so $F_1^{k,x}$ is also a best response.

Proof of (iii): Suppose $z_1 = Z(x)$. Given $F_1^{k,x}$, $U_2(0^1, F_1^{k,x}) = U_2(\infty, F_1^{k,x})$, and so any $F_2^{k,x}(0^1) \in [0,1]$ is a best response. Moreover, for any $F_2^{k,x}(0^1) \in [0,1]$, we have $U_1(0^2, F_2^{k,x}) \geq U_1(0^1, F_2^{k,x})$ as required.

Proof of (*2*): Assume that $0 < k < \min\{K^*(x), T(x)\}$. Negotiator 1's reputation at k^1 be $Z(x)$ only if $F_1^{k,x}(0) > 0$. Therefore, it must be that $c_2^{k,x} = 1$ by Proposition 1, yielding the functional form of $F_2^{k,x}$. Solving $\hat{z}_1(k^1) = Z(x)$ yields $F_1^{k,x}(t)$.

Proof of (*3*): Assume that $0 < k = K^*(x) < T(x)$. If $F_1^{k,x}(0) > 0$, then $F_2^{k,x}(0) = 0$, and so $\hat{z}_2(k^1) < Z(x) < \hat{z}_1(k^1)$ by (1) of Proposition 1, contradicting (3) of Proposition 1. Thus, is must be that $F_1(0) = 0$, implying $F_1^{k,x}$ as required. Because $F_2^{k,x}(k^1) \leq 1$ by Proposition 1, it must be that $c_2^{k,x} \geq z_2 e^{\lambda_2(x)k^1}$ as required.

Proof of (*4*): Assume that $K^*(x) < k < T(x)$. Because $K^*(x) < k$ and $F_1^{k,x}$ is strictly increasing by Proposition 1, it must be that $Z(x) < \hat{z}_1(k^1)$. The last condition and (3) of proposition 1 require $F_2^{k,x}(k^1) = 1$, implying $F_2^{k,x}(t)$ as required by Proposition 1. Since $c_2^{k,x} < 1$, Proposition 1 requires $c_1^{k,x} = 1$, implying $F_1^{k,x}$.

Proof of (*5*): The proof immediately follows from Hendricks, Weiss and Wilson (1988) and the proof of Lemma 1 in Abreu and Gul (2000). ■

Proof of Theorem 2. Given Theorem 1, in equilibrium after any history $(x, k) \in \mathbb{D} \times \overline{\mathbb{R}}_+$ of the game G where $k > 0$, negotiators are indifferent between conceding at time 0 and waiting for some time $t \leq k^2$ and then conceding at time t. Thus, $U_1\big(t, F_2^{k,x}\big) = U_1\big(0, F_2^{k,x}\big)$ whenever $t \in [0, k]$. Namely,

$$U_1\big(t, F_2^{k,x}\big) = x_1(1-z_2)F_2^{k,x}(0) + (1-x_2)\Big[1-(1-z_2)F_2^{k,x}(0)\Big] \equiv u_1^{k,x}$$

for all $0 \leq t \leq k^1$. Thus,

$$U_1^e\big(F_1^{k,x}, F_2^{k,x}\big) = (1-z_1)u_1^{k,x}.$$

Therefore, Theorem 1 implies that Negotiator 1's ex-ante payoff is as follows:

(A) If $k=0$, then
(A.1) If $z_1 \geq Z(x)$, then $U_1^e(F_1^{k,x}, F_2^{k,x}) = x_1(1-z_1)(1-z_2) + (1-x_2)z_2(1-z_1)$.
(A.2) If $z_1 < Z(x)$, then $U_1^e(F_1^{k,x}, F_2^{k,x}) = (1-x_2)(1-z_1)$.
(B) If $k < \min\{K^*(x), T(x)\}$, then $U_1^e(F_1^{k,x}, F_2^{k,x}) = (1-x_2)(1-z_1)$
(C) If $K^*(x) \leq k < T(x)$, then $U_1^e(F_1^{k,x}, F_2^{k,x}) = x_1(1-z_1)\left(1-z_2 e^{\lambda_2(x)k}\right) + (1-x_2)(1-z_1)z_2 e^{\lambda_2(x)k}$.
(D) If $T(x) \leq k$, then $U_1^e(F_1^{k,x}, F_2^{k,x}) = x_1(1-z_1)\left(1-z_2 e^{\lambda_2(x)T(x)}\right) + (1-x_2)(1-z_1)z_2 e^{\lambda_2(x)T(x)}$.

Because $x_1 > 1-x_2$ and all the payoffs are convex combination of x_1 and $1-x_2$, it is easy to verify that payoff in **(C)** decreases with k, is higher than that of **(B)** and **(D)** for all $k \in [0, T(x)]$, lower than that of **(A.1)**, and higher than **(A.2)**. Moreover, payoff in **(D)** is higher than that of **(B)** and **(A.2)**. Thus, if $Z(x) \leq z_1$, then optimal deadline k is such that $k=0$, and if $\underline{Z}(x) \leq z_1 \leq Z(x)$, then optimal k satisfies $k = K^*(x)$ because the payoff in **(A.1)** is redundant. Finally, if $z_1 < \underline{Z}(x)$, then $K^*(x) > T(x)$, so payoff in **(C)** is also redundant. In this case, optimal k is such that $k \geq T(x)$. ■

Proof of Theorem 3. Define the set

$$D(G) = \left\{\alpha_1 \in (0,1) \middle| \underline{Z}(\alpha_1, \alpha_2) \leq z_1 \text{ for all } \alpha_2 \in [1-\alpha_1, 1)\right\}.$$

Then, set

$$\alpha^* \equiv \sup D(G).$$

Note that α^* is well-defined because $D(G)$ is a nonempty and bounded set: For any values of z_1 and z_2, there always exists α_1 sufficiently close to 0 so that $\underline{Z}(\alpha_1, \alpha_2)$ is close to 0 and less than z_1.

By Theorems 1 and 2, given demands $\alpha = (\alpha_1, \alpha_2)$, Negotiator i's lowest ex-ante payoff in equilibrium is $\underline{u}_i \equiv (1-z_i)(1-\alpha_j)$. Moreover, because concession game strategies satisfy $F_1^{k,\alpha}(0)F_2^{k,\alpha}(0) = 0$, if Negotiator i's equilibrium payoff is more than $\underline{u}_i$, then j's payoff must be $\underline{u}_j$. Furthermore, if $z_1 < \underline{Z}(\alpha)$, then Negotiator 1's ex-ante payoff is $\underline{u}_1$ regardless of her deadline announcement in Stage 2. However, if $z_1 \geq \underline{Z}(\alpha)$, then Negotiator 1's equilibrium payoff is more than $\underline{u}_1$.

Therefore, demands $\alpha \in \mathbb{D}$ cannot be part of equilibrium if $z_1 < \underline{Z}(\alpha)$ and there is some $\alpha_1' \in [1-\alpha_2, 1)$ for Negotiator 1 such that $z_1 \geq \underline{Z}(\alpha_1', \alpha_2)$. Note that for any $\alpha_2 \in (0,1)$ there is always some α_1' that is sufficiently close to $1-\alpha_2$ such that $z_1 \geq \underline{Z}(\alpha_1', \alpha_2)$ holds. Thus, in equilibrium we should never have $z_1 < \underline{Z}(\alpha)$. Symmetrically, demands $\alpha \in \mathbb{D}$ are not part of equilibrium if $z_1 \geq \underline{Z}(\alpha)$ and there is $\alpha_2' \in [1-\alpha_1, 1)$ for Negotiator 2 such that $z_1 < \underline{Z}(\alpha_1, \alpha_2')$. Hence, equilibrium demand for Negotiator 1 must be the supremum of the set $D(G)$ in equilibrium, and because Negotiator 2's ex-ante payoff is $\underline{u}_2 < 1-\alpha^*$, her best response is to demand $(1-\alpha^*)$ and finish the game in Stage 1. ■

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