Take-It-or-Leave-It Offers in Negotiations: Behavioral Types and Endogenous Deadlines*

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May 2022

Abstract

This paper investigates the effects of *take-it-or-leave-it* offers on rational negotiators' equilibrium behaviors. A negotiator who makes a take-it-or-leave-it offer bluffs that her offer is final (*commitment threat*), and depending on her position, that she is ready to leave the bargaining table (*exit threat*). These are credible threats in the current model because there is a possibility that negotiators may be behavioral types, who for some reason are constrained to commit to the conditions of the standing offers and threats.

JEL Codes: C72, C78; D82

Keywords: Bilateral bargaining, deadline effect, reputational bargaining, war of attrition, continuous-time games, behavioral types, exit threat, endogenous deadline.

1. Introduction

In episode 14 (season 6) of the famous TV series *House*, the main story revolves around the last eight hours of a critical negotiation between Lisa Cuddy (the dean of Medicine) and a contract negotiator from Atlantic Net Insurance to renew a contract. Dr. Cuddy and the contract negotiator have been arguing about the contract for eight months, and that day, Dr. Cuddy lays it all on the line. When they meet at 8:30 a.m., Dr. Cuddy makes her final offer that she agrees to capitated structure but wants a 12% increase in rates. The contract negotiator refuses Dr. Cuddy's offer immediately. Then she tells him that this is the hospital's final offer, and he has until 3:00 p.m. to agree, or she will make a public announcement that they are no longer accepting Atlantic Net. At noon, Dr. Cuddy tracks down the CEO of Atlantic Net at

*Earlier version of this paper was circulated with the title ""Take It or Leave It" Offers: Obstinacy and Endogenous Deadlines in Negotiations." I thank Emin Karagözoğlu and all seminar participants at Bilkent University for many useful discussions and suggestions, Sabancı University Economics Group for their mentoring, support and hospitality, and the Scientific and Technological Research Council of Turkey (TUBITAK) for their funding and generous support. I especially thank Mustafa Emre Demirel, whom I jointly started this project in 2012, for his valuable collaboration.

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lunch and confronts him about the contract. He blows her off and tells her that her tactic will not work. The negotiator from Atlantic Net returns around 2pm and offers an 8% increase as their final offer, but Dr. Cuddy declines and wants the full 12%. After a few stressful hours of waiting, the story ends with a good news. The negotiator revisits Dr. Cuddy just before her announcement and tells that the company has agreed to her 12% proposal.

Negotiators commonly use *take-it-or-leave-it* offers as a final strategic maneuver in order to push their rivals towards more acceptable terms. It usually begins with a milder threat such as "this is my final offer," and if it does not work, escalates to a higher level: "take this one or I am calling off the negotiation." If the bargainers' offers do not get closer to each other after a lengthy delay, followed by offers and counteroffers, then a take-it-or-leave-it offer could be an optimal strategy to speed things up. This paper, however, is not interested in explaining what leads negotiators to a point where they threaten each other so vigorously. The focus is investigating the impacts of these bluffs *after* they are made.

Subtext of a take-it-or-leave-it offer contains a threat of not making further concessions (i.e., *commitment threat*), and depending on the negotiator's position, a threat of leaving the bargaining table (i.e., *exit threat*). For example, both firms and unions would make a take-it-or-leave-it offer in collective bargaining, but unions' threats usually follow an announcement for a strike. Threats are effective if they are credible, so I follow a reputational approach. That is, I study a stylized four-stage, infinite-horizon, continuous-time bargaining game that I adapt from the reputational bargaining literature (see, for example, Özyurt 2015a & 2015b, Abreu and Gul 2000, and Kambe 1999). The novel twist is that a negotiator can bluff about ending the negotiation, by announcing a deadline. There are four defining features of the model. First, two negotiators bargain over the division of a unit surplus and begin with announcing their demands (final offers). Second, Negotiator 1 declares a deadline for the game, if the demands are incompatible. Third, each negotiator faces some small uncertainty that her opponent may be a behavioral type, who acts on her threats. Fourth, the negotiation phase adopts a war of attrition protocol, during which each agent chooses between conceding (yielding the opponent), waiting for the opponent's concession, and leaving the game. Potentially small but positive uncertainty regarding opponent's true type provides strong incentive for negotiators to build reputation on their commitments, affecting their equilibrium behavior.

This exercise is important for three reasons. First, commitment and exit threats are studied separately in the bargaining literature, but the current work is the first attempt, to the best of my knowledge, combining the two. Second, ability to exit the game creates endogenous deadline effect, which is absent in the reputational bargaining literature.[1] Third, equilibrium behaviors (strategies) are essentially unique, and provide interesting testable hypothesis and explanations for some widely noted observations in the experimental literature (see Section 3 for details).

[1]Compte and Jehiel (2002) allow negotiators to "*exit*" (i.e., *end the game without a concession*) at any time they want; but exit is an action that is more favorable than concession. In Özyurt (2015 a&b) a negotiator (buyer) can go back and forth between two opponents (sellers). By doing so she builds her reputation on obstinacy, and the value of her outside option. Nevertheless, the buyer does not have an option to exit the game. Fanning (2016) studies deadline effect in a standard reputational bargaining framework, but deadline is exogenous for negotiators.

2. The Framework

Two negotiators, 1 and 2, bargain over the division of a unit surplus. Negotiators discount time, and $r_i \in \mathbb{R}_{++}$ denotes rate of time preference for Negotiator $i \in \{1,2\}$. **Reputational bargaining game**, denoted by G, is modeled by the following four-stage, infinite-horizon, and continuous-time game:

All four stages begin, and the first three stages end at time 0. No discounting applies between these four stages. In Stage 1 each negotiator independently and simultaneously announces a share of the unit surplus $x_i \in (0,1)$, denoting Negotiator i's final offer (or demand). If these demands are compatible (i.e., $x_1 + x_2 \leq 1$), then the game is over in Stage 1, and one of the two divisions of surplus $(x_1, 1-x_1)$ or $(1-x_2, x_2)$ is implemented with a probability of 1/2 each. If the demands are incompatible (i.e., $x_1 + x_2 > 1$), then the game moves to the next stage. In Stage 2 Negotiator 1 sends a cheap talk message, and announces a deadline: Time that she intends to leave the bargaining table and end the game.[2] Negotiator 2 does not take any action in this stage. In Stage 3 each negotiator learns whether she is allowed to continue the game; Negotiator i is replaced with a "*behavioral type*" with a probability $z_i \in (0,1)$, and allowed to stay in the game with probability $1 - z_i$. In Stage 4 (still time 0) the following continuous-time concession game begins: At any given time $t \geq 0$, a negotiator either leaves the bargaining table (i.e., exits the game), or accepts her opponent's Stage 1 demand, or waits for her concession. If a negotiator concedes or leaves the negotiation table, then the game ends.

Because the concession game is in continuous time, there may occur some measure-theoretic pathologies associated with negotiators exiting the game and conceding at a given time. I resolve such potential issues in the manner introduced by Abreu and Pearce (2007). In particular, for any time $t \geq 0$ of Stage 4, corresponding to the "conventional time" t, I suppose two logically consecutive stages, t^1 and t^2, of time t. No discounting applies between these "two stages." A negotiator can concede to her opponent at both stages, t^1 and t^2, but can exit the game only at the first stage, t^1, of time t.

Negotiators are risk neutral. If the game ends at time $t \geq 0$ with Negotiator i's concession, then the payoffs to Negotiator i and j are $(1 - x_j)e^{-r_i t}$ and $x_j e^{-r_j t}$, respectively. If i concedes and j exits in Stage t^1 of time t, then the game ends with i's concession. In case of simultaneous concession, one of the two divisions of surplus, $(x_1, 1-x_1)$ or $(1-x_2, x_2)$, is implemented with a probability of 1/2 each. If both negotiators wait forever or a negotiator exits the game, then both receive a payoff of zero.[3] A negotiator who is replaced with a behavioral type also receives a payoff of zero. Negotiators maximize the expected discounted values of their shares. The entire structure of the reputational bargaining game G is common knowledge.

[2] Stage 2 announcement is a cheap talk message in the sense that the game does not end at the announced deadline, and Stage 4 continues forever (i.e., beyond the announced deadline) if both negotiators choose to do so.

[3] Exiting the game is an inefficient outcome, for much the same reasons as strikes in collective bargaining. It represents committing oneself to an irreversible course, also known as *burning bridges*.

Strategies of the Behavioral Types: A behavioral type simply commits to the announcements of her predecessor. That is, behavioral type of Negotiator 1 never concedes, and she exits the game at the announced deadline. Behavioral type of Negotiator 2 never concedes and never exits the game.

Strategies of the (Rational) Negotiators: A negotiator's strategy must describe her choice of demand and her decision about when (if ever) to concede to her opponent and exit the game, given demands and deadline. Negotiator 1's strategy must also describe her choice of a deadline, given demands.

More formally, Stage 1 strategy of Negotiator i is a pure action $x_i \in (0,1)$. Let $x \equiv (x_1, x_2)$ denote negotiators' Stage 1 demands. Stage 2 strategy of Negotiator 1 is also a pure action, denoted by $K_x \in [0,\infty] \equiv [0,\infty) \cup \{\infty\}$, where $K_x = \infty$ is interpreted as Negotiator 1 not announcing a deadline. Given the history of the first two stages, summarized by K_x, Stage 4 strategy for Negotiator i is a right-continuous distribution function $F_i^{K_x} : [0,\infty] \to [0,1]$, representing the probability of Negotiator i conceding to Negotiator j by conventional time t (inclusive of both Stage t^1 and t^2 of time t). Therefore, $F_i^{K_x}(t^1)$ denotes the probability of Negotiator i conceding to j by time t inclusive of Stage t^1 of time t, and $F_i^{K_x}(t) = F_i^{K_x}(t^2)$.

Negotiator i makes zero payoff when she leaves the bargaining table. However, she can make a positive payoff in any equilibrium by conceding to her opponent since $x_j \in (0,1)$. Thus, negotiators never exit the game in equilibrium. For this reason and for simplicity, I ignore negotiators' exit strategies. Furthermore, if Negotiator 1 announces K_x in Stage 2, then her behavioral type will exit the game at time K_x^1, if the game reaches this point. Therefore, Negotiator 1's type (i.e., rationality) will be common knowledge at time K_x^2 if she remains in the game, in which case the concession game turns into a standard war of attrition game with one sided incomplete information. It is a well-established result in the reputational bargaining literature (see, for example, Lemma 1 of Fanning 2016) that a negotiator who is known to be rational must concede immediately in equilibrium, when facing a possibly behavioral type. Therefore, I suppose, without loss of generality, that negotiators never concede beyond the deadline; namely, $F_i^{K_x}(t) = F_i^{K_x}(K_x)$ for all $t \geq K_x$ and $i \in \{1,2\}$.

Payoffs: Given the negotiators' strategies, summarized by $F_1^{K_x}$ and $F_2^{K_x}$, let $B_i^{K_x} : [0,\infty] \to [0, 1-z_i]$ denote Negotiator j's belief about Negotiator i accepting x_j and finishing the game prior to time t. Thus, $B_i^{K_x}(t) = (1-z_i)F_i^{K_x}(t)$ since behavioral types never concede.

Negotiator i's **interim payoff** —payoff after learning that she will continue the game— of conceding to Negotiator j at time $t \leq K_x$, conditional that the game has not yet over, is

$$U_i(t, F_j^{K_x}) = (1-x_j)\left[1 - B_j^{K_x}(t)\right] e^{-r_i t} + x_i \int_0^t e^{-r_i y} dB_j^{K_x}(y) + \frac{1}{2}(1 + x_i - x_j)\left[B_j^{K_x}(t) - B_j^{K_x}(t^-)\right] e^{-r_i t}, \quad (1)$$

where $B_j^{K_x}(t^-) = (1-z_j)\lim_{y \nearrow t} F_j^{K_x}(y)$. Note that the last term drops if Negotiator j's strategy, $F_j^{K_x}$ is continuous. Negotiator i's **ex-ante payoff** —payoff before learning Stage 3 outcome—

is

$$U_i^e(F_i^{K_x}, F_j^{K_x}) = (1 - z_i) \int_0^\infty U_i(y, F_j^{K_x}) dF_i^{K_x}(y). \tag{2}$$

Equilibrium refers to Perfect Bayesian Nash equilibrium.

Like Crawford (1982), Kambe (1999), Wolitzky (2012), and Ellingsen and Miettinen (2014), probability of commitment (i.e., z_i's) is independent of the announcements. There are two motivations for adopting this approach. First, many real-world negotiations agree with this approach: Negotiators may (have to) commit to their demands or threats, regardless of how unpleasant it may be, depending on how events unfold during negotiation. For example, a state leader may commit to her announcements if revoking her commitments turns out to be a very costly action (such as cost of losing face or credibility of her rhetoric), and the leader may not know the size of these costs for a fact before seeing the reactions of her constituency. The second motivation has a technical rationale. Abreu and Gul (2000) interpret behavioral types as irrational players that are born with their commitments. Given this interpretation, if negotiator i is rational and demanding x_i, then this is her strategic choice. If she is a behavioral type, then she merely declares the demand corresponding to her type. One can easily extend this approach to the current setup. However, this would lead to a substantially complex model when there is a large set of types.

3. Overview of the Results and Comparison with the Empirical Findings

This section summarizes important aspects of the equilibrium and compares them with the empirical and experimental findings in the literature. Important to note that experiments that I cite below are not designed specifically to test the model of this paper. For a start, these experiments consider exogenous deadlines, yet it is endogenous in my model. For this reason, perhaps, Cramton and Tracy (1992) is the most relevant study. They consider union contract negotiations, where unions correspond to Negotiator 1 and strikes —threats that are inefficient for all parties and that only union can make— correspond to the exit threat in my model. Cramton and Tracy (1992) call expiration date of the old contracts by deadline. These dates correspond to the beginning of Stage 4 (time 0) in my model because expiration date is the earliest time that a union can start a strike. A deadline in my model, however, corresponds the beginning of a strike. In light of this mapping between these two frameworks, their data indicates that daily settlement rate has a peak right after the expiration date (i.e., time 0) and then decreases gradually with time. After the first week, the weekly settlement rate from dispute is roughly constant at 11 %. These observations agree with the equilibrium behaviors, and I discuss this in detail under items (4) and (7) below.

To minimize the notation, I assume throughout this section that negotiators are ex-ante identical; namely $z_i \equiv z$ and $r_i = r$ for $i = 1, 2$. Equilibrium outcome is unique and efficient (Theorem 3): Negotiators 1 and 2 announce compatible demands α^* and $1 - \alpha^*$, respectively, and end the game in Stage 1. Equilibrium share α^* does not have a closed-form solution, but numerical simulations indicate that Negotiator 1 significantly benefits from having the option to announce a deadline, though it is a cheap talk message and she does not execute this threat in equilibrium. Assuming that negotiators are ex-ante identical, equal share of the unit surplus would be an equilibrium, absent Stage 2 (see Kambe 1999). However, α^* takes values 0.788, 0.699, and 0.638 (approximately) when the probability of commitment, z, is 0.4, 0.1, and 0.01, respectively. Though its convergence is slower than z, α^* converges 0.5 in the limit when z approaches zero. Namely, effects of the exit threat fade away with exogenous commitment probability, z.

Uniqueness and efficiency of the equilibrium outcome is not the interesting result per se because calculating α^* is an impossible task even for experienced rational players in a controlled lab environment. For this reason, the rest of this section summarizes how negotiators behave on "off the equilibrium path"; in cases where mistakes occur, and negotiators choose incompatible demands in Stage 1.

I start with summarizing the equilibrium deadline (see Theorem 2). Assuming that Stage 1 demands are incompatible, namely the excess demand $E_x \equiv x_1 + x_2 - 1$ is positive, there are three parameters, z, z_x^H and z_x^L, that play key role in determining the optimal deadline: The first of these parameters, z, is the negotiators' commitment probability, which is exogenous to the model. The other two are endogenously determined by the negotiators' demands as follows:

$$z_x^H = \frac{E_x}{x_2} \quad \text{and} \quad z_x^L = z_x^H z^{R_x},$$

where the *ratio of residual demands*, R_x, is defined by $R_x = \frac{1-x_1}{1-x_2}$. Note that z_x^H measures Negotiator 2's share of the excess demand. It is easy to verify that $0 < z_x^L < z_x^H < 1$ because demands are incompatible. If z is less than the lower threshold, z_x^L, then announcing an "effective" deadline hurts Negotiator 1. In this case, she announces a deadline K_x big enough so that the concession game strategies would be the same if she had announced it as ∞. Such deadlines are called **ineffective deadlines**. If z is higher than the upper threshold, z_x^H, then Negotiator 1 behaves very aggressively, and announces $K_x = 0$. Namely, she threatens to turn the concession game into an ultimatum game. Finally, for the intermediate values of z, the optimal deadline (i.e., K_x^*) is uniquely determined by

$$K_x^* = \frac{\ln\left(z_x^H/z\right)}{\lambda_x^1},$$

where $\lambda_x^1 = \frac{r(1-x_1)}{E_x}$ is Negotiator 1's instantaneous concession rate (i.e., hazard rate). K_x^* increases with Negotiator 2's time preferences. That is, Negotiator 1 prefers to impose more time pressure on her rival (by announcing a shorter deadline) as Negotiator 2 is more impatient, and less time pressure (by announcing a longer deadline) as 2 is more patient. To sum, equilibrium deadline effectively takes three values; $0, K_x^*$, and ∞ depending on how z is compared relative to z_x^L and z_x^H.

Assuming that excess surplus E_x is positive and the announced deadline K_x is positive but not ineffective, negotiators' Stage 4 equilibrium behaviors satisfy the following (see Theorem 1):

1. **Delay in agreements:** Negotiators play a mixed strategy, where they are indifferent between conceding and waiting at all times before the deadline. Although negotiators discount time, they prolong the agreement all the way to the announced deadline with a positive probability because they can build their reputation on their commitments. Higher reputation (by waiting) compensates the opportunity cost of delay, giving the negotiators enough incentive to postpone the agreement with a positive probability.

2. **Likelihood of disagreements:** If the game reaches the announced deadline, then rational Negotiator 1 concedes, and behavioral type exits the game immediately. In either case, Stage 4 certainly ends no later than the deadline. However, rational Negotiator 2 may allow the game end with a disagreement. This may happen only if Negotiator 1 **undershoots** the target deadline (i.e., she announces a deadline that is less than the optimal level, K_x^*). The **likelihood of disagreement** (i.e., the probability that Negotiator 2 allows the game end with a disagreement) decreases with the deadline. That is, as the announced deadline approaches zero, likelihood of disagreement approaches one. This finding is supported by Karagözoğlu and Kocher (2019), but contradicts with some of the earlier research on time pressure effects on bargaining (see, for example, Roth et al. 1988; and De Dreu et al. 2000). These earlier papers claim that high time pressure (by an earlier deadline) induces lower resistance to conceding, and so individuals under high time pressure might care less about their own position and are more willing to compromise. This does not hold in the model because undershooting the target deadline is an "immature bluffing," signaling that Negotiator 1 is not likely to act on her threat, and so, Negotiator 2 is more likely to call her bluff and less willing to compromise.

3. **Initial shock and deadline effects:** Concession game strategies are continuous and have no atoms on the interval $(0, K_x)$. Each negotiator concedes to her opponent with a constant hazard rate λ_x^i. Concession with a positive probability (i.e., atom) is possible only at time 0 (**initial concession**) or at the announced deadline K_x (**final concession**).

4. **Initial shock effect:** Negotiator 1 makes initial concession with a positive probability if and only if she undershoots the target deadline, K_x^*. On the other hand, Negotiator 2 makes initial concession with a positive probability if and only if the announced deadline is more than K_x^* (i.e., when Negotiator 1 **overshoots** the target deadline). Therefore, initial shock effect always exists and is due by either negotiator, depending on whether Negotiator 1 overshoots or undershoots the target deadline. Existence of strong initial shock effect is supported by Cramton and Tracy (1992) and Güth et al. (2005). Probability of initial concession decreases with deadline. Although this relationship is mostly monotonic, there may be jumps at the optimal deadline, K_x^*.

5. **Deadline effect (last-moment agreements):** Negotiator 1 always makes a final concession with a positive probability, if the game ever reaches this point. This observation agrees with a large body of experimental and empirical evidence on deadline effects (see, for example, Güth et al. 2005 and Roth et al. 1988). Negotiator 2 never makes a final

concession. Therefore, observable deadline effects are always due by Negotiator 1.

6. **Intensity of the deadline effect:** Probability of final concession (or the deadline effect) does not change with the deadline if Negotiator 1 undershoots the target. If she overshoots, however, then this probability decreases with the announced deadline, as summarized below:

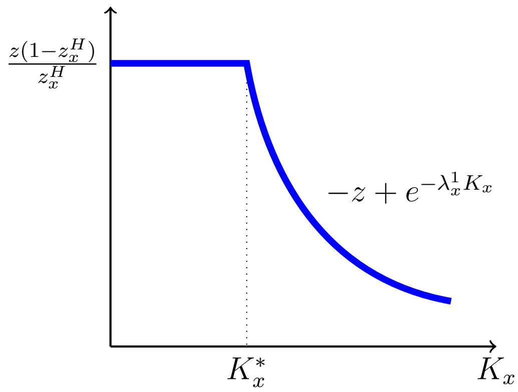

Figure 1: Probability of deadline effect as a function of the announced deadline

This finding agrees with Karagözoğlu and Kocher (2019) where they report that frequency of last-moment agreements is higher in high time pressure treatment than in low time pressure treatment.

To understand the reason of this behavior, some background information is necessary: Negotiator i's instantaneous concession rate is constant, regardless of the announced deadline K_x, and equal to the hazard rate λ_x^i. Namely, the growth rate of her equilibrium concession strategy, $F_i^{K_x}$, must be constant. Moreover, Negotiator 1's concession strategy always reaches one in equilibrium; namely she does not let the game end with a disagreement. If Negotiator 1 undershoots or overshoots the deadline, then she must adjust her strategy because she did something suboptimal. Adjusting strategy means Negotiator 1 must be sure that $F_1^{K_x}$ reaches probability one before the deadline without any discontinuity on $(0, K_x)$ or any change in its growth rate. Therefore, if Negotiator 1 needs to make "adjustments" to her equilibrium concession behavior, then she has only two options; adjusting the initial concession rate or the final concession rate.

If Negotiator 1 undershoots the deadline, then she knows that she made an "immature bluff," and her opponent will call her bluff with a positive probability. Therefore, backing down earlier by conceding at the beginning of the concession game is better for her than waiting and backing down at the very last moment. As a result of this, frequency of last-moment agreements does not change with the announced deadline. However, if Negotiator 1 overshoots the deadline, then she is not worried about immature bluffing, and so she does not make an immediate concession. In this case, however, frequency of last-moment agreements decreases with the announced deadline because she has more "opportunity" to concede before the deadline.

7. **Settlement rate:** For an outsider, who observes the announced deadline and demands, but not the negotiators' types, settlement rate (i.e., probability that Stage 4 ends by a concession of a negotiator) *at time* $t \in (0, K_x)$ is the sum of the instantaneous concession rates, $\lambda_x^1 + \lambda_x^2$. Therefore, settlement rate has two peaks, one at time zero and the other at the deadline, though the latter is not as high as the former, and flat otherwise.

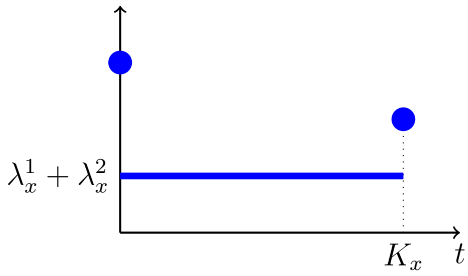

Figure 2: Settlement rate at time t.

These observations agree with the findings of Cramton and Tracy (1992), given the transformation discussed at the beginning, and Güth et al. (2005), where they show that settlement rate has one peak at the beginning and another at the end, and mostly flat in between.

4. Formal Results

Negotiator i's behavior in Stage 4, given final offers $x = (x_1, x_2)$ and deadline K_x, is a probability distribution over concession times,

$$F_i^{K_x}(t) = Pr(\text{Negotiator } i \text{ concedes prior to time } t).$$

Let $\lambda_x^i(t)$ be her instantaneous concession (or hazard) rate at time t. By definition, it satisfies

$$\lambda_x^i(t) = \frac{dB_i^{K_x}(t)/dt}{1 - B_i^{K_x}(t)} = \frac{(1 - z_i)dF_i^{K_x}(t)/dt}{1 - (1 - z_i)F_i^{K_x}(t)}. \tag{3}$$

Concession game has no equilibrium in pure strategies. In a mixed strategy equilibrium, negotiators mix between conceding and waiting. Negotiator j is indifferent between conceding at time t and waiting for an infinitesimal period Δ and then conceding at time $t + \Delta$ if and only if

$$(1 - x_i)e^{-r_j t} = x_j e^{-r_j t}\lambda_x^i(t)\Delta + [1 - \lambda_x^i(t)\Delta](1 - x_i)e^{-r_j(t+\Delta)},$$

where $\lambda_x^i(t)\Delta$ indicates the probability that Negotiator i concedes during Δ. Solving the last equation for $\lambda_x^i(t)$ and taking its limit as Δ approaches 0 yields a time independent hazard rate:

$$\lambda_x^i = \frac{r_j(1 - x_i)}{x_1 + x_2 - 1}. \tag{4}$$

Using (3) and integrating up the hazard rate (4) yields

$$F_i^{K_x}(t) = \frac{1}{1-z_i}\left(1 - c_i^{K_x} e^{-\lambda_i t}\right),$$

where $c_i^{K_x} = 1 - F_i^{K_x}(0)$. Therefore, Negotiator i's Stage 4 strategy is constrained by the fixed hazard rate λ_x^i. The announced deadline changes the horizon of the game and the initial concession probability, $F_i^{K_x}(0)$. For any $x = (x_1, x_2) \in (0,1)^2$, define

$$E_x = x_1 + x_2 - 1, \quad z_x^H = \frac{E_x}{x_2}, \quad z_x^L = z_x^H z_2^{\lambda_1/\lambda_2}, \quad K_x^* = \frac{\ln(z_x^H/z_1)}{\lambda_x^1},$$

and

$$T_x^0 = \min\left\{\frac{-\ln z_1}{\lambda_x^1}, \frac{-\ln z_2}{\lambda_x^2}\right\}.$$

Next result summarizes Stage 4 strategies.

Theorem 1. *In equilibrium, after any history of the game G where $E_x > 0$, negotiators' Stage 4 strategies satisfy the following:*

1. *If $K_x = 0$, then*
   - *(i) $F_1^{K_x}(0^1) = 0$, $F_1^{K_x}(0^2) = 1$, and $F_2^{K_x}(0^1) = 1$ whenever $z_x^H < z_1$,*
   - *(ii) $F_2^{K_x}(0^1) = F_2^{K_x}(0^2) = 0$, $0 \leq F_1^{K_x}(0^1) \leq 2\left(\frac{z_x^H - z_1}{z_x^H(1-z_1)}\right)$, and $F_1^{K_x}(0^2) = 1$ whenever $z_1 < z_x^H$, and*
   - *(iii) $F_1^{K_x}(0^1) = 0$, $F_1^{K_x}(0^2) = 1$, $0 \leq F_2^{K_x}(0^1) \leq 1$, and $F_2^{K_x}(0^2) = F_2^{K_x}(0^1)$ whenever $z_1 = z_x^H$.*
2. *If $0 < K_x < \min\{K_x^*, T_x^0\}$, then*
   - *(i) $F_1^{K_x}(t) = \frac{1}{1-z_1}(1 - \frac{z_1}{z_x^H} e^{\lambda_x^1(K_x^1 - t)})$ whenever $t \in [0, K_x^1]$, and $F_1^{K_x}(K_x) = 1$, and*
   - *(ii) $F_2^{K_x}(t) = \frac{1}{1-z_2}(1 - e^{-\lambda_x^2 t})$ whenever $t \in [0, K_x^1]$, and $F_2^{K_x}(K_x) = F_2^{K_x}(K_x^1)$.*
3. *If $K_x^* < T_x^0$ and $K_x = K_x^*$, then*
   - *(i) $F_1^{K_x}(t) = \frac{1}{1-z_1}(1 - e^{-\lambda_x^1 t})$ whenever $t \in [0, K_x^1]$, and $F_1^{K_x}(K_x) = 1$,*
   - *(ii) $F_2^{K_x}(t) = \frac{1}{1-z_2}(1 - c_2^{K_x} e^{-\lambda_x^2 t})$ where $c_2^{K_x} \in \left[z_2\left(\frac{z_x^H}{z_1}\right)^{\lambda_x^2/\lambda_x^1}, 1\right]$ whenever $t \in [0, K_x^1]$, and $F_2^{K_x}(K_x) = F_2^{K_x}(K_x^1)$.*
4. *If $K_x^* < T_x^0$ and $K_x^* < K_x < T_x^0$, then*
   - *(i) $F_1^{K_x}(t) = \frac{1}{1-z_1}(1 - e^{-\lambda_x^1 t})$ whenever $t \in [0, K_x^1]$, and $F_1^{K_x}(K_x) = 1$, and*
   - *(ii) $F_2^{K_x}(t) = \frac{1}{1-z_2}(1 - z_2 e^{\lambda_x^2(K_x^1 - t)})$ whenever $t \in [0, K_x^1]$, and $F_2^{K_x}(K_x) = F_2^{K_x}(K_x^1)$.*
5. *Finally, if $T_x^0 \leq K_x$, then $F_i^{K_x}(t) = \frac{1}{1-z_i}(1 - z_i e^{\lambda_x^i(T_x^0 - t)})$ whenever $t \in [0, T_x^0]$, and $F_i^{K_x}(t) = 1$ whenever $t \geq T_x^0$.*

Given the equilibrium strategies, negotiators build their reputation simply by waiting. Negotiator i's reputation at time t, denoted by $\hat{z}_x^i(t)$, is the conditional probability that she is the behavioral type (conditional on the event that she does not concede before time t) and it is calculated by the Bayes' rule as follows:

$$\hat{z}_x^i(t) = \frac{z_i}{z_i + (1 - z_i)(1 - F_i^{K_x}(t))}.$$

In equilibrium, Negotiator 1 announces as early a deadline as possible with the constraint that she must have sufficient time to build her reputation so that $\hat{z}_x^1(K_x^1) \geq z_x^H$ holds. Negotiator 1's reputation at the deadline cannot fall short of z_x^H because if it does, then Negotiator 2 would prefer to wait rather than concede at or around the deadline, contradicting that mixed strategy equilibrium requires indifference between these two actions at all times. The bigger the time pressure Negotiator 1 imposes on her rival (by announcing a shorter deadline), the earlier her rival would concede, and thus, the more surplus (in ex-ante terms) Negotiator 1 could extract. However, Negotiator 1's reputational concerns prevent him to announce a very short deadline. In conclusion, Negotiator 1's reputational concerns give rise to a trade-off between the demands, x, and the length of the deadline she announces, K_x, and this trade-off is summarized by the inequality above. The next result formally summarizes all this.

Theorem 2. *In equilibrium, after any history of the game G where $E_x > 0$, Negotiator 1's deadline announcement, K_x, satisfies the following:*

1. *$K_x = 0$ whenever $z_1 \geq z_x^H$,*
2. *$K_x = K_x^*$ whenever $z_x^L \leq z_1 < z_x^H$, and*
3. *$K_x \in \left[-\frac{\ln z_2}{\lambda_x^2}, \infty\right]$ whenever $z_1 < z_x^L$.*

Negotiators never choose extreme demands in equilibrium, and the next result characterizes Stage 1 strategies.

Theorem 3. *In any equilibrium of the game G, Negotiator 1 demands α^*, satisfying*

$$\alpha^* = \sup\left\{x_1 \in (0,1) \middle| z_{(x_1,x_2)}^L \leq z_1 \text{ for all } x_2 \in [1 - x_1, 1)\right\},$$

Negotiator 2 demands $1 - \alpha^*$, and the game ends in Stage 1.

5. Concluding Remarks and Related Literature

Commitment and exit threats are studied, to some extent, separately in the bargaining literature. Schelling (1966) points out the potential benefits of commitment in strategic and dynamic environments and asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. Crawford (1982), Muthoo (1996), and Ellingsen and Miettinen (2008) follow this approach and show that commitment can be rationalized in equilibrium if revoking commitments is costly. The bargaining models in these papers are one-shot simultaneous-move games. Myerson (1991), Kambe (1999), and Abreu and Gul (2000) follow a reputational approach: Parallel to Kreps and Wilson (1982) and Milgrom and Roberts (1982), commitments are modeled as behavioral types that exist in society so that rational players can mimic if they like to do so.[4]

The exit threat is also studied in the bargaining literature. Among many others, Osborne and Rubinstein (1990), Vislie (1988), Shaked (1994), and Ponsati and Sakovics (1998 & 2001) model exit as the ability of opting out of negotiation and receiving an outside option. On the other hand, Fershtman and Seidmann (1993), Ma and Manove (1993), and Ponsati (1995), for example, model exit as a predetermined deadline for the negotiations. The treatment of the exit threat in the current paper has resemblance to both approaches in the sense that announcing an exit time creates a deadline effect, and the ability of choosing exit time provides strategic advantage for much the same reason as the ability of opting out does. Two important difference of the current paper, however, is that (1) rational negotiators do not have to exit the game, unless it is what they want to, and could continue negotiation beyond the deadline, and (2) opting out is an inefficient outcome for all, so it is never optimal to exit.

Fudenberg and Tirole (1986), Chatterjee and Samuelson (1987), and Ponsati and Sakovics (1995) study war of attrition (WOA) games with two-sided uncertainty. Kambe (1999) and Abreu and Gul (2000) take a step forward and add a pre-play to standard WOA games, where the negotiators simultaneously choose their demands, which determine their strategies in the WOA phase. The current paper adds additional layer to these papers by allowing one player to announce a deadline, at which point her behavioral type may end the game.

Appendix

I begin with proving the following result, which I extensively use to prove Theorem 1.

Proposition 1. *In equilibrium, after any history of the game G where $E_x > 0$ and $K_x \in (0, T_x^0)$, negotiators' Stage 4 strategies satisfy the following:*

1. $F_i^{K_x}(t) = \frac{1}{1-z_i}(1 - c_i^{K_x} e^{-\lambda_x^i t})$ *whenever* $t \in [0, K_x^1]$, *where* $c_i^{K_x} \in [z_i, 1]$ *with* $(1 - c_1^{K_x})(1 - c_2^{K_x}) = 0$.
2. $F_1^{K_x}(K_x^2) = 1$ *and* $F_2^{K_x}(K_x) = F_2(K_x^1)$.
3. *Negotiator 1's reputation reaches* z_x^H *at time* K_x^1, *namely* $\hat{z}_i^{K_x}(K_x^1) = z_x^H$, *whenever* $\hat{z}_2^{K_x}(K_x^1) < 1$.

Proof of Proposition 1. Assume that $E_x > 0$ and $K_x \in (0, T_x^0)$, and suppose $F_1^{K_x}$ and $F_2^{K_x}$ are part of an equilibrium. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and are analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

[4] Abreu and Sethi (2003) supports the existence of behavioral types from an evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such behavioral types is not compatible with evolutionary stability in bargaining environments.

Lemma A.1. *If $F_i^{K_x}$ is constant on some interval $[t,t'] \subseteq [0,K_x^1)$, then $F_j^{K_x}$, where $i,j \in \{1,2\}$ and $j \neq i$, is also constant over $[t,t'+\eta]$ for some $\eta > 0$.*

Lemma A.2. *$F_1^{K_x}$ and $F_2^{K_x}$ do not have a mass point over $(0,K_x^1)$.*

Lemma A.3. *$F_1^{K_x}(0)F_2^{K_x}(0) = 0$.*

Proof of (*1*): There is no interval (t',t'') with $0 \le t' < t'' < K_x^1$ such that both $F_1^{K_x}$ and $F_2^{K_x}$ are constant. Assume on the contrary that $t^* < K^1$ is the supremum of the upper bounds of t'''s such that both $F_1^{K_x}$ and $F_2^{K_x}$ are constant. By Lemma A.1, $F_j^{K_x}$ is constant on $(t',t^*+\eta)$ for some $\eta > 0$ because $F_i^{K_x}$ is constant on (t',t^*). Thus, both $F_1^{K_x}$ and $F_2^{K_x}$ are constant on this later interval, contradicting the definition of t^*. Therefore, $F_1^{K_x}$ and $F_2^{K_x}$ must be strictly increasing over $[0,K^1]$. Lemma A.2 implies the strategies are continuous over $[0,K^1)$. To prove that they are continuous on $[0,K_x^1]$ as well, suppose for a contradiction that $F_2^{K_x}$ has a jump at time K_x^1. Then Negotiator 1 prefers to wait for some time before K_x^1 and concede at time K_x^2. However, it contradicts the fact that $F_1^{K_x}$ is strictly increasing over $[0,K_x^1]$. Likewise, $F_1^{K_x}$ cannot have a jump at time K_x^1. To prove the last claim, suppose for a contradiction that $F_1^{K_x}(K_x^1) - F_1^{K_x}((K_x^1)^-) = p > 0$. Then, we have

$$U_2(K_x^1-\Delta, F_1^{K_x}) = (1-x_1)(1-B_1^{K_x}(K_x^1-\Delta))e^{-r_2(K_x^1-\Delta)} + x_2\int_0^{K_x^1-\Delta} e^{-r_2 y}dB_1^{K_x}(y),$$
$$U_2(K_x^1, F_1^{K_x}) = \left[\frac{1}{2}(1+x_2-x_1)p + (1-x_1)(1-B_1^{K_x}(K_x^1))\right]e^{-r_2K_x^1} + x_2\int_0^{K_x^1} e^{-r_2 y}dB_1^{K_x}(y).$$

Therefore, $U_2(K_x^1, F_1^{K_x}) - U_2(K_x^1-\Delta, F_1^{K_x}) > 0$ for small values of Δ because this difference is equal to $o_1(\Delta) + o_2(\Delta) + \frac{1}{2}p(1+x_2-x_1)e^{-r_2K_x^1}$ where $o_1(\Delta) = x_2\int_{K_x^1-\Delta}^{K_x^1} e^{-r_2y}dB_1^{K_x}(y)$, $o_2(\Delta) = (1-x_1)[(1-B_1^{K_x}(K_x^1)) - (1-B_1^{K_x}(K_x^1-\Delta))e^{r_2\Delta}]$ and both $o_1(\Delta)$ and $o_2(\Delta)$ approach 0 as Δ approaches 0. In conclusion, if $F_1^{K_x}$ has a jump at time K_x^1, then Negotiator 2 prefers to wait for some time $[K_x^1-\Delta, K_x^1)$, where $\Delta > 0$ is small, and then concede at time K_x^1, contradicting that $F_2^{K_x}$ is not constant over $[0,K_x^1]$. Hence, $F_1^{K_x}$ and $F_2^{K_x}$ must be continuous on $[0,K^1]$. Given the functional form of $U_i(t,.)$, $U_i(t,F_j^{K_x})$ must be continuous as well.

Then, it follows that $D^i \equiv \{t | U_i(t,F_j^{K_x}) = \max_{s\in[0,K_x^1]} U_i(s,F_j^{K_x})\}$ is dense in $[0,K_x^1]$. Hence, $U_i(t,F_j^{K_x})$ is constant for all $t \in [0,K_x^1]$. Consequently, $D^i = [0,K_x^1]$. Therefore, $U_i(t,F_j^{K_x})$ is differentiable as a function of t. The differentiability of $F_1^{K_x}$ and $F_2^{K_x}$ follows from the differentiability of the utility functions on $[0,K_x^1]$. Differentiating the utility functions and applying the Leibnitz's rule yields the functional forms as required.

Proof of (*2*): Equilibrium strategy $F_2^{K_x}$ dictates that Negotiator 2's rationality will never be common knowledge before the game ends. However, Negotiator 1's type will be revealed at time K_x^2 if the game ever comes to this point. In equilibrium, Negotiator 2 will never concede to a rational negotiator after time K_x^1, so Negotiator 1 will concede and finish the game at time K_x^2. Hence, it must be that $F_1^{K_x}(K_x^2) = 1$ and $F_2^{K_x}(K_x) = F_2(K_x^1)$.

Proof of (*3*): Assume that $\hat{z}_x^2(K_x^1) < 1$. In equilibrium, Negotiator 2 is indifferent between conceding and waiting at time K_x^1 because $F_2^{K_x}$ is continuous on $[0,K_x^1]$. If she concedes at time K_x^1, then her instantaneous payoff will be $1-x_1$. If she waits instead, then her expected payoff will be $(1-\hat{z}_x^1(K_x^1))x_2$ by (*2.*) above. These two payoffs are equal if and only if $\hat{z}_x^1(K_x^1) = z_x^H$, as required. ■

Proof of Theorem 1. Take any history of the game G where $E_x > 0$. Assume that $F_i^{K_x}$ is Stage 4 equilibrium strategy for $i = 1, 2$.

Proof of (*1*): Assume $K_x = 0$. It must be that

$$
\begin{aligned}
U_1(0^1, F_2^{K_x}) &= F_2^{K_x}(0^1)(1-z_2)\left(\frac{1+x_1-x_2}{2}\right) + z_2(1-x_2) + (1-z_2)(1-F_2^{K_x}(0^1))(1-x_2),\\
U_1(0^2, F_2^{K_x}) &= (1-z_2)F_2^{K_x}(0^1)x_1 + [(1-z_2)(1-F_2^{K_x}(0^1)) + z_2](1-x_2),\\
U_2(0^1, F_1^{K_x}) &= F_1^{K_x}(0^1)\left(\frac{1+x_2-x_1}{2}\right)(1-z_1) + z_1(1-x_1) + (1-z_1)(1-F_1^{K_x}(0^1))(1-x_1), \text{and}\\
U_2(\infty, F_1^{K_x}) &= (1-z_1)F_1^{K_x}(0^1)x_2 + (1-z_1)(1-F_1^{K_x}(0^1))x_2.
\end{aligned}
$$

where $U_2(\infty, F_1^{K_x})$ denote Negotiator 2's expected payoff of never conceding.

Proof of (*i*): Suppose $z_x^H < z_1$. Given $F_1^{K_x}$, $F_2^{K_x}$ form a best response because $U_2(\infty, F_1^{K_x}) \leq U_2(0^1, F_1^{K_x})$ iff $(1-z_1)x_2 \leq z_1(1-x_1) + (1-z_1)(1-x_1)$ iff $z_x^H \leq z_1$. Similarly, given $F_2^{K_x}$, $F_1^{K_x}$ forms a best response because $U_1(0^2, F_2^{K_x}) = x_1(1-z_2)+z_2(1-x_2) > U_1(0^1, F_2^{K_x}) = z_2(1-x_2)+(1-z_2)(\frac{1+x_1-x_2}{2})$ iff $x_1 + x_2 > 1$. To establish uniqueness, first observe that $U_1(0^2, F_2^{K_x}) > U_1(0^1, F_2^{K_x})$ (or $F_1^{K_x}(0^1) = 0$) whenever $F_2^{K_x}(0^1) > 0$. However, if $F_1^{K_x}(0^1) = 0$, then Negotiator 2 prefers to concede at Stage 0^1 of time 0. Therefore, we must show it is never the case that $F_2^{K_x}(0^1) = 0$. Suppose, for a contradiction, that $F_2^{K_x}(0^1) = 0$. It must be that $U_2(F_1^{K_x}) \geq U_2(0^1, F_1^{K_x})$ iff $F_1^{K_x}(0^1) \leq 2(\frac{z_H - z_1}{z_H(1-z_1)})$, which never holds since $z_1 > z_H$.

Proof of (*ii*): Suppose $z_1 < z_x^H$. Given the range of $F_1^{K_x}$, $U_2(\infty, F_1^{K_x}) \geq U_2(0^1, F_1^{K_x})$, and so $F_2^{K_x}$ is a best response. Likewise, given $F_2^{K_x}$, $U_1(0^1, F_2^{K_x}) = U_1(0^2, F_2^{K_x}) = 1 - x_2$, and so $F_1^{K_x}$ is also a best response.

Proof of (*iii*): Suppose $z_1 = z_x^H$. Given $F_1^{K_x}$, $U_2(0^1, F_1^{K_x}) = U_2(\infty, F_1^{K_x})$, and so any $F_2^{K_x}(0^1) \in [0, 1]$ is a best response. Moreover, for any $F_2^{K_x}(0^1) \in [0, 1]$, we have $U_1(0^2, F_2^{K_x}) \geq U_1(0^1, F_2^{K_x})$ as required.

Proof of (*2*): Assume that $0 < K_x < \min\{K_x^*, T_x^0\}$. Negotiator 1's reputation at K_x^1 be z_H only if $F_1^{K_x}(0) > 0$. Therefore, it must be that $c_2^{K_x} = 1$ by Proposition 1, yielding the functional form of $F_2^{K_x}$. Solving $\hat{z}_x^1(K_x^1) = z_x^H$ yields $F_1^{K_x}(t)$.

Proof of (*3*): Assume that $0 < K_x = K_x^* < T_x^0$. If $F_1^{K_x}(0) > 0$, then $F_2^{K_x}(0) = 0$, and so $\hat{z}_x^2(K_x^1) < z_x^H < \hat{z}_x^1(K_x^1)$ by (1) of Proposition 1, contradicting (3) of Proposition 1. Thus, is must be that $F_1(0) = 0$, implying $F_1^{K_x}$ as required. Because $F_2^{K_x}(K_x^1) \leq 1$ by Proposition 1, it must be that $c_2^{K_x} \geq z_2 e^{\lambda_x^2 K_x^1}$ as required.

Proof of (*4*): Assume that $K_x^* < K_x < T_x^0$. Because $K_x^* < K_x$ and $F_1^{K_x}$ is strictly increasing by Proposition 1, it must be that $z_x^H < \hat{z}_x^1(K_x^1)$. The last condition and (3) of proposition 1 require $F_2^{K_x}(K_x^1) = 1$, implying $F_2^{K_x}(t)$ as required by Proposition 1. Since $c_2^{K_x} < 1$, Proposition 1 requires $c_1^{K_x} = 1$, implying $F_1^{K_x}$.

Proof of (5): The proof immediately follows from Hendricks, Weiss and Wilson (1988) and the proof of Lemma 1 in Abreu and Gul (2000). ■

Proof of Theorem 2. Given Theorem 1, in equilibrium after any history of the game G where $E_x > 0$ and $K_x > 0$, negotiators are indifferent between conceding at time 0 and waiting for some time $t \leq K_x^2$ and then conceding at time t. Thus, $U_1\big(t, F_2^{K_x}\big) = U_1\big(0, F_2^{K_x}\big)$ whenever $t \in [0, K_x]$. Namely,

$$U_1\big(t, F_2^{K_x}\big) = x_1(1-z_2)F_2^{K_x}(0) + (1-x_2)\Big[1-(1-z_2)F_2^{K_x}(0)\Big] \equiv u_1^{K_x}$$

for all $0 \leq t \leq K_x^1$. Thus,

$$U_1^e\big(F_1^{K_x}, F_2^{K_x}\big) = (1-z_1)u_1^{K_x}.$$

Therefore, Theorem 1 implies that Negotiator 1s' ex-ante payoffs are as follows:

(A) If $K_x = 0$, then
(A.1) If $z_1 \geq z_x^H$, then $U_1^e(F_1^{K_x}, F_2^{K_x}) = x_1(1-z_1)(1-z_2) + (1-x_2)z_2(1-z_1)$.
(A.2) If $z_1 < z_x^H$, then $U_1^e(F_1^{K_x}, F_2^{K_x}) = (1-x_2)(1-z_1)$.
(B) If $K_x < \min\{K_x^*, T_x^0\}$, then $U_1^e(F_1^{K_x}, F_2^{K_x}) = (1-x_2)(1-z_1)$
(C) If $K_x^* \leq K_x < T_x^0$, then $U_1^e(F_1^{K_x}, F_2^{K_x}) = x_1(1-z_1)\left(1-z_2e^{\lambda_x^2 K_x}\right) + (1-x_2)(1-z_1)z_2e^{\lambda_x^2 K_x}$.
(D) If $T_x^0 \leq K_x$, then $U_1^e(F_1^{K_x}, F_2^{K_x}) = x_1(1-z_1)\left(1-z_2e^{\lambda_x^2 T_x^0}\right) + (1-x_2)(1-z_1)z_2e^{\lambda_x^2 T_x^0}$.

Because $x_1 > 1-x_2$ and all the payoffs are convex combination of x_1 and $1-x_2$, it is easy to verify that payoff in **(C)** decreases with K_x, is higher than that of **(B)** and **(D)** for all $K_x \in [0, T_x^0]$, lower than that of **(A.1)**, and higher than **(A.2)**. Moreover, payoff in **(D)** is higher than that of **(B)** and **(A.2)**. Thus, if $z_x^H \leq z_1$, then optimal deadline announcement for Negotiator 1 is $K_x = 0$, and if $z_x^L \leq z_1 \leq z_x^H$, then optimal deadline announcement is $K_x = K_x^*$ because the payoff in **(A.1)** is redundant. Finally, if $z_1 < z_x^L$, then $K_x^* > T_x^0$, so payoff in **(C)** is also redundant. In this case, optimal deadline announcement for Negotiator 1 is any $K_x \geq T_x^0$. ■

Proof of Theorem 3. Define the set

$$D(G) = \left\{x_1 \in (0,1)\middle| z_x^L \leq z_1 \text{ for all } x_2 \in [1-x_1, 1)\right\}.$$

Then, set

$$\alpha^* \equiv \sup D(G).$$

Note that α^* is well-defined because $D(G)$ is a nonempty, bounded set: For any values of z_1 and z_2, there always exists x_1 sufficiently close to 0 so that z_x^L is close to 0, and so less than z_1.

By Theorems 1 and 2, given demands $x = (x_1, x_2)$, Negotiator i's lowest ex-ante payoff in equilibrium is $\underline{u}_i \equiv (1-z_i)(1-x_j)$. Moreover, because concession game strategies satisfy $F_1^{K_x}(0)F_2^{K_x}(0) = 0$, if Negotiator i's equilibrium payoff is more than $\underline{u}_i$, then j's payoff must be $\underline{u}_j$. Furthermore, if $z_1 < z_x^L$, then Negotiator 1's ex-ante payoff is $\underline{u}_1$ regardless of her deadline announcement in Stage 2. However, if $z_1 \geq z_x^L$, then Negotiator 1's equilibrium payoff is more than $\underline{u}_1$.

Therefore, demands (x_1, x_2) cannot be part of equilibrium if $z_1 < z_x^L$ and there is $x_1' \in [1-x_2, 1)$ for Negotiator 1 such that $z_1 \geq z^L_{(x_1', x_2)}$. Note that for any $x_2 \in (0,1)$ there is always a x_1 that is sufficiently close to $1 - x_2$ such that $z_1 \geq z^L_{(x'_1,x_2)}$ holds. Thus, in equilibrium we should never have $z_1 < z^L_x$. Symmetrically, demands (x_1, x_2) are not part of equilibrium if $z_1 \geq z^L_x$ and there is $x'_2 \in [1 - x_1, 1)$ for Negotiator 2 such that $z_1 < z^L_{(x_1,x'_2)}$. Hence, equilibrium demand for Negotiator 1 must be the supremum of the set $D(G)$ in equilibrium, and because Negotiator 2's ex-ante payoff is $\underline{u}_2 < 1 - \alpha^*$, her best response is to demand $(1 - \alpha^*)$ and finish the game in Stage 1. ■

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